

# The Impact of Using Geometers' Sketchpad on Malaysian Students' Achievement and Van Hiele Geometric Thinking

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*This quasi-experimental research was designed to examine the impact of using geometers' sketchpad on geometry achievement and the level of van Hiele geometric thought among Form Three students in one of the secondary schools in Perak, Malaysia. A total of 65 Form Three students from the school were chosen for this research. The treatment group (N=32) underwent the lessons using the geometers' sketchpad for ten weeks. At the same time the control group (N=33) was taught by the traditional approach. The finding of this study about the effects of Geometer's Sketchpad and the van Hiele model will be useful to mathematics teachers and educators.*

**Key words:** Geometers' Sketchpad; Geometry; Achievement; Level of geometric thinking; van Hiele

The *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and other important literature in the area of reform in mathematics education (Mathematical Sciences Education Board, 1990; National Research Council, 1989) call for change in emphasis and content in geometry at all levels. The introduction of geometers' sketchpad gave mathematics educators opportunities to help students not only solve geometry problems but also to discuss, justify, and help improve thinking.

Learning of geometry is formally introduced in the Malaysian primary mathematics curriculum. The emphasis in geometry increases as students progress to secondary education, where about forty percent of the sixty topics in the five-year secondary mathematics curriculum comprises geometry content (Malaysian Ministry of Education, 1998). Geometry is a unifying theme to the entire mathematics curriculum and as such is a rich source of visualization for arithmetical, algebraic, and statistical concepts. For example,

geometric regions and shapes are useful for development work with the meaning of fractional numbers, equivalent fractions, ordering of fractions, and computing with fractions (Sanders, 1998, p. 20).

The Topics of triangles and quadrilaterals are covered from primary to secondary schools. Thus, the students are able to identify triangles and quadrilateral since Year One. However, according to Noraini Idris (1999) from her clinical interviews conducted on 13 and 14-year-old students from public school in Selangor, Malaysia, it was found that the words “square” and “rectangle” were not part of their normal vocabulary.

As students are introduced to increasingly abstract concepts and exposed to relational operators such as equal, congruent, and similar, their mental models of the relationships among concepts require continual restructuring (Battista et. al., van de Walle, 2001, Olive, 2000). From the perspective of the van Hiele model of the development of geometric thought, the student moves from observing and identifying the figure to a recognition of its properties, to understanding the interrelationships of the properties of the figures and the axiomatic system within which they are placed (Usiskin, 2003).

Geometers’ Sketchpad provides a flexibly structured mathematics laboratory that supports the investigation and exploration of concepts at a representational level, linking the concrete to the abstract. Many students are not able to comprehend what their mathematics teachers teach especially on the topic of geometry because mathematics content is taught with the intention of finishing the syllabus and preparing for examinations. Little regard is given to how well the students understand geometrical concepts. On the topic of geometry, students encounter difficulties in applying what they have learned as they were not given enough time to understand the geometry concepts. Instead they were just memorizing the concepts.

### **Statement of Problem**

Learning geometry may not be easy, and a large number of the students fail to develop an adequate understanding of geometry concepts, geometry reasoning, and geometry problem solving skills (Battista & Borrow, 1997; Elchuck, 1992; Noraini, 1999). The lack of understanding in learning geometry often causes discouragement among the students, which invariably will lead to poor performance in geometry. A number of factors have been put forward to understand why geometry learning is difficult – geometry language,

visualization abilities, and ineffective instruction (Cangelosi, 1996; Noraini, 2006).

Poor reasoning skills are also another area of concern among secondary school students. Many are unable to extract necessary information from given data and many more are unable to interpret answers and make conclusions. Traditional approaches in learning geometry emphasize more on how much the students can remember and less on how well the students can think and reason. Thus learning becomes forced and seldom brings satisfaction to the students. This study was designed to explore the effects of Geometers' Sketchpad on van Hiele geometric thought.

## **Conceptual Framework**

### **The van Hiele Model of Learning in Geometry**

The conceptual framework of the van Hiele Model of learning geometry was built on a model consisting of five levels of thought development in geometry. The van Hieles were greatly concerned about the difficulties their students encountered with secondary school geometry. The van Hiele model has three main components: insight, phases of learning, and thought levels (Hoffer, 1983; Usiskin, 2003). Many of the ideas for the insight and structure component were "borrowed from Gestalt theory" (van Hiele, 1986, p.5) "Insight exists when a person acts in a new situation adequately and with intention. The Gestalt psychologist and I say the same thing with different words" (van Hiele, p. 24).

The second component of the van Hiele model, the phases of learning, describes the phases through which students' progress in order to attain the next higher levels of thinking. Basically these phases constitute an outline for organizing instruction. Van Hiele (1986) identifies five phases in this learning process and gives an example of the stages in the study of the rhombus:

- 1) In the first phase, instruction should begin with an inquiry phase in which materials lead children to explore and discover certain structures. That is, through working with material presented to them, students become acquainted with the structure of the material, such as, examining examples and non-examples of geometric concepts. For example, a certain figure is demonstrated, it is called "rhombus." The pupils are shown other geometrical figures and are asked if they also are rhombuses.

- 2) In the second phase, given by teacher or made by themselves, tasks are formed with different relations of the network. Students actively engage in exploring objects (for example, folding, measuring) so as to encounter the principal connections of the network of relations that is to be formed. For example, students are asked to fold the rhombus on its axes of symmetry and observe what happens to the diagonals and the angles.
- 3) In the third phase, explicitation, pupils become conscious of the relations, they try to express them as words; they learn the technical language accompanying the subject matter. For example, the pupils exchange their ideas about the properties of a rhombus.
- 4) 4. In the fourth phase, free orientation, pupils learn through general tasks to find their own way in the network of relations. For example, some vertices and sides of a rhombus are given by position. The whole rhombus has to be constructed.
- 5) 5. In the fifth phase, integration, pupils build an overview of all they have learned of the subject, of the newly formed network of relations now at their disposal. For example, the properties of a rhombus are summed up and memorized (pp. 53-54).

The transition from one level to the next is a learning “process that has to be done by the pupils themselves” (van Hiele, 1986, p. 62). Teachers can give guidance to the students during this complicated exercise: “Transition from one level to the following is not a natural process; it takes place under the influence of a teaching-learning program” (p.50). The teachers’ choice of lessons and activities is critical in the transition from one level to the next. In this manner, teachers help students find ways to ascend to the next higher level. During the transition, van Hiele considers discussion to be the most important part of the teaching-learning process and without learning a new language, the transition is impossible.

The third component of the van Hiele model grew out of the concern the van Hieles felt when their geometry students repeatedly encountered difficulties with parts of the subject matter even after being given various explanations. Their joint interest in wanting to improve teaching outcomes led to the development of a theoretical model involving five levels of geometric thinking.

According to the van Hieles, the learner, assisted by appropriate instructional experiences, passes through the following five levels, where the

learner cannot achieve one level of thinking without having passed through the previous levels. It is clear that throughout those phases of learning, the teacher has various roles: planning tasks, directing students' attention to geometric qualities of figures, introducing terminology and engaging students in discussions using these terms and encouraging explanations.

Hoffer (1981) describes the van Hiele levels of learning in geometry in the following manner:

Level 1: Recognition. The student identifies, names, compares and operates on geometric figures (e.g., triangles, angles, or intersecting) according to their appearance

Level 2: Analysis. The student analyzes figures in terms of their components and relationship among components and discovers properties/rules of a class of shapes empirically (e.g. by folding, measuring, using grid or diagram).

Level 3: Ordering. The student logically interrelates previously discovered properties / rules by giving or following informal arguments.

Level 4: Deduction. The student proves theorems deductively and established interrelationships among networks of theorems.

Level 5: Rigor. The students establishes theorems in different postulation systems and analyzes/compares these systems (p.35)

The van Hieles made certain observations about the general nature of these levels of thinking and their relationship. Hiele (1959) notes: "At each level there appears in an extrinsic way that which was intrinsic at the preceding level. At level I, figures were in fact determined by their properties, but someone thinking at level I is not aware of these properties" (p. 202). Since his initial work, van Hiele has focused his attention and in-depth descriptions to Level 1 through 4, for it is at the lower levels that most geometry students function. It is more likely that a thorough understanding of the lower levels will lead to improving the teaching and learning of geometry. The higher levels are easily over-valued and have only theoretical value. The main characteristics of the levels are: (1) The levels have a hierarchic arrangement through which the person moves sequentially. (2) Moving from one level to the next is more a result of a learning process rather than a result of age or maturation. (3) The learning process which leads to a higher level is distinguished by various phases of learning. (4) Each level has a unique language, set of symbols, and network of relations joining these symbols. (5) What appears in an explicit manner at one level is intrinsic at the preceding level. (6) A person reasoning at the higher level cannot be understood by

another person at a lower level. (7) Material taught above a person's level may be reduced to a lower level by that person.

Van Hiele (1959) states that the levels are "characterized by differences in objects of thought" (p.14). For example, at level 1, the objects of thought are geometric figures. At level 2, the student operates on certain objects, namely, classes of figures, which are products of level 1 activities and discovers properties for these classes. At level 3, these properties become the objects that the student acts upon, yielding logical orderings of these properties. At level 4, the ordering relations become the objects on which the student operates and at level 5, the objects of thought are the foundation of these ordering relations.

The transition from one level to the next is a learning "process that has to be done by the pupils themselves" (p. 62). Teachers can give guidance to the students during the exercises. Transition from one level to the following is not a natural process; it takes place under the influence of a teaching-learning program. The teachers' choice of lessons and activities is critical in the transition from one level to the next. In this manner, teachers help students find ways to ascend to the next higher level.

Thus, this study investigated the van Hiele levels for secondary school students, to examine whether van Hiele levels of geometric thought relate to student's performance in geometry, and to evaluate the instructional activities developed for improving van Hiele levels of geometric reasoning.

### **Objective of the Study**

This purpose of this study is to investigate the effect of using the geometer's sketchpad on students' achievement and the van Hiele geometric thinking. Specifically, the study intends to:

- 1) Investigate the effects of geometers' sketchpad on geometry achievement.
- 2) Examine the effects of geometers' sketchpad on van Hiele level.
- 3) Find out the perceptions of students towards the use of geometers' sketchpad.

### **Research Questions**

Specifically, this study will attempt to answer the following research questions: 1) Is there any significant effect of using geometers' sketchpad in

students' achievement of geometry? 2) Is there any significant effect of using geometers' sketchpad in students' van Hiele level? 3) What are the students' perceptions about using geometers' sketchpad?

## **Procedure**

### *Research Design*

The study was a quasi-experimental nonequivalent control group design. In this study the researcher did not assign subjects randomly to treatments. Pre- and post-measures were administered: (1) changes in van Hiele Geometric scores and (2) changes in geometry achievement.

### *Sample*

The participants in this study were students in one of the secondary school in Perak, Malaysia. The students in this study were from a middle social-economic status. The average ages of the students were between 14 to 15 years. There were five classes in Form Three. According to the principals, each class was assigned with mixed ability –high, average and low. After discussion with the principal and teachers, two intact classes were identified for this study. One class was assigned to be the experimental group and the other class was assigned to be the control group. Both groups had comparable socio-economic and ethnic backgrounds as well as comparable mathematics grades according to the teachers' grade book. The sample studied consisted of 65 students. Of the 65 in the total sample, 32 students of the treatment group underwent the lessons using the Geometers' Sketchpad for ten weeks. At the same time the control group was taught by a traditional approach.

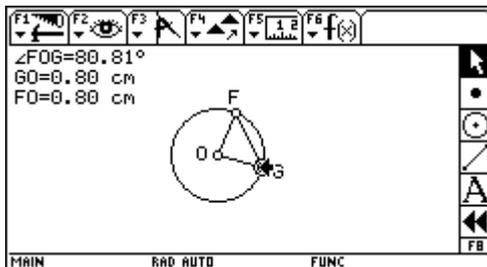
### *Instrumentation*

The students' van Hiele levels of geometric thinking were assessed by van Hiele Geometry Test (VHGT). The test modified by Usiskin (1982) to assess van Hiele level based on the van Hiele's descriptions of the five levels of geometric thinking was adapted for this study. The test-retest reliability of van Hiele levels are .71. Pre- and posttests of geometry test and VHGT were administered to both the experimental and control groups to compare the students' geometry achievement and van Hiele levels of Geometric Thinking.

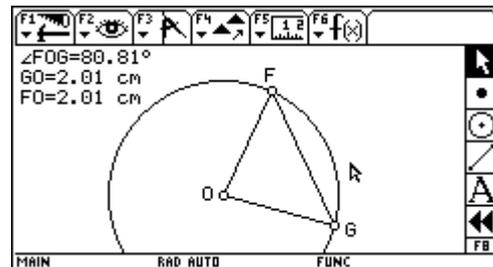
The questionnaire was administered to investigate the students' response towards the use of Geometers' Sketchpad on learning of geometry.

### *Instructional Materials*

The goal of instructional activities in this study was to improve van Hiele level and achievement in geometry. The researcher selected and/or designed the instructional activities for the teachers to use during the ten weeks treatment. The activities involve investigating geometric objects and properties to deepen students' geometric concepts. Different instructional materials were used for the experimental and control groups. Experimental groups used instructional activities based on Geometers' Sketchpad as in Figure 1 and 2 below.



*Figure 1. Drawing of Triangle*



*Figure 2. Enlargement of Triangle*

The activities allowed the students to explore, investigate, discover, reflect, and visualize the geometrical concepts based on Geometer's Sketchpad to help the students in upgrading their mathematical understanding and attain higher mathematical achievement.

The students were first given an introductory lesson on the basic use of the Geometers' Sketchpad. The prepared activities were used in the following lessons and the teachers would teach more Geometers' Sketchpad skills as the lessons progressed. In the control groups, the topics were taught with the use of a mathematics textbook and without the use of Geometers' sketchpad.

### *Data Analysis*

Descriptive statistics of frequencies was computed using the S Plus 6 statistical package to compare the differences of the student van Hiele Geometric Thinking of both the control and experimental groups.

## Results

### Effects of Geometer's Sketchpad on Geometry Achievement

The result as shown in Table 2 show that in the pre-achievement test, there is no significant difference between experimental and control group at  $p < .05$ . In the posttest, the control group exhibited a mean of 13.08 whilst the experimental group showed a mean of 19.65. The computed t-value between the posttest of the control and the experimental groups showed a value of 2.78 with  $p = .02$ . The result showed that there is a significant difference between the control and experimental groups.

*Table 1*  
**Mean, Standard Deviation and t-values for Experimental and Control Groups**

Test	Groups	Mean	SD	t-value	p-value
Pretest	Control	6.71	1.19	0.788	0.43
	Experimental	6.69	1.18		
Posttest	Control	13.08	6.78	2.78	0.02
	Experimental	19.65	5.97		

### Effects of Geometer's Sketchpad-based instructions on van Hiele Level

The subjects were sorted by group and by change in van Hiele level from pre- to posttest. After posttest, all subjects either remained at the same level or were assigned to a level one or two above their pretest level. Table 1 showed students' changes in level from pretest to posttest for experimental and control groups.

*Table 2*  
**Comparison of Students' Pre- and Posttest van Hiele Levels (Experimental, Control)**

Pre	Posttest				
	0	1	2	3	4
0	(3, 14)	(8, 6)			
1		(4, 6)	(5, 1)	(1, 0)	
2			(4, 5)	(2, 0)	

3	(3, 1)
4	(2, 0)

As shown in Table 2, the ordered pair (3, 14) under the level 0 columns indicates that there were three students in the experimental group and fourteen students in the control group who remained the same from pre- and posttest. The ordered pair (5, 1) under level 2 indicates that five students in the experimental group and one in the control group moved from level 1 to level 2.

To answer the question whether subjects in the experimental group using Geometers’ Sketchpad achieved significantly greater change in van Hiele levels compared to subjects in the control group who did not use Geometers’ Sketchpad, a Kruskal-Wallis test was performed on the data. Table 3 provides the result of Kruskal-Wallis Test.

*Table 3*  
**Rank Change in van Hiele Levels by Group**

Group	Sum of scores	Expected Under Ho	SD under Ho	Mean score
Experimental	378.95	413.97	45.72	4.80
Control	309.17	345.85	43.58	3.63

$\chi^2 = 18.72$

**The result of the test with  $\chi^2 = 18.72$ ,  $df = 1$ , indicates a significant difference ( $p < .01$ ) between treatment and control groups on subjects’ change in rank on van Hiele levels from pretest to posttest.**

**Perceptions of Students towards the Use of Geometers’ Sketchpad (GSP)**

Thirty three form two students completed a form with the questions as shown in Table 2. The scaled score is calculated based on 5 – strongly agree, 4 – agree, 3 – not sure, 2 – disagree, and 1 – strongly disagree.

*Table 4*  
**Students’ Survey Results on the Usage of Geometers’ Sketchpad**

Item	5	4	3	2	1	Min
1. Geometers’ Sketchpad helped me in understanding the topics better.	12	1	5	5	0	3.91

2. GSP help to visualize	10	13	7	3	0	3.89
3. I am able to interact with my teacher & friends	12	13	6	1	1	3.92
4. I feel confident about trying a new problem.	11	14	6	2	0	4.01
5. Geometers' Sketchpad make me feel comfortable learning geometry.	12	13	5	2	1	3.94

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As shown in Table 3, most of the students showed positive reactions towards the use of Geometers' Sketchpad. Students felt confident about trying a new problem on the Geometers' Sketchpad with a min 4.01. Students felt that Geometers' Sketchpad made them comfortable learning mathematics, min 3.94.

### **Discussion and Conclusion**

From the results obtained, a number of implications can be forwarded in the interest of improving geometry teaching in the classroom. Firstly, the significant differences in geometry achievement of the experimental groups as compared to the control groups indicate that the geometer's sketchpad shows promising implications for the potential of using the Geometers' Sketchpad in teaching geometry at the secondary school level. The result of this study is consistent with the Sanders (1998) study which reported that the addition of dynamic geometry software in geometric construction had increased her students' interest in geometry as well as enhancing their understanding. This observation can therefore encourage classroom teachers and even curriculum developers of the potential of the geometer's sketchpad as effective tool in learning geometry.

Van Hiele-based instructional materials and the use of Geometers' Sketchpad played a special role in helping students to progress within a level or to a higher level (Elchuck, 1992; Engebretsen, 1997; Glass et al, 2001; Schattschneider & King, 2001, Noraini Idris, 2006). Tasks that involved a variety of environments in which the concepts were embodied, such as drawing, identifying and exploration revealed modes of reasoning about specific concepts that the researcher could identify with confidence. According to the van Hiele model, each leaning period builds on and extends the thinking of the proceeding level. This is significant for teachers in selecting and sequencing instructional activities in accordance with the model.

Effective learning occurs as students actively experience the objects of study in appropriate contexts of geometric thinking and as they engage in discussion and reflection using the language of the learning period . Awareness and knowledge of students' van Hiele levels can be a useful asset and tool to the geometry teacher in the classroom. The significant improvement of geometry achievement on account of the specially prepared van Hiele-based instructional and Geometers' Sketchpad used in this study also suggest that there is a need to provide more interactive and hands-on learning activities for geometry learning at the lower secondary school level.

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