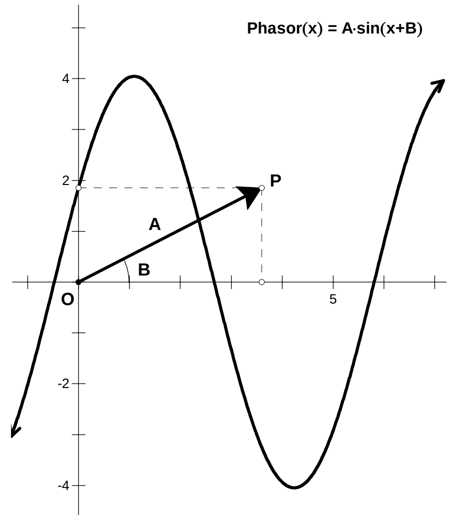
When you plug your computer into the wall, it runs using electric power supplied by the outlet. This power is in the form of alternating current (ac).

The variable x represents time, in units related to the period of the wave.

If you measure either the voltage or the current of this ac electricity, the result is a sinusoidal wave with a particular magnitude, A, and a phase shift, B, that can be written as $A \cdot \sin(x + B)$. The expression $x + B$ is called a phase of the wave.

At any given time, the y-component of the phasor is equal to the value of the function $A \cdot \sin(x + B)$.

A *phasor* for this wave is defined as a rotating vector with magnitude A and initial angle B. The vector rotates with the same period as the sine function. Note the difference between a vector and a phasor: a vector has a constant direction, while the direction of a phasor changes over time, since it depends on x.

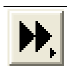


Electrical engineers use *phasor diagrams* to calculate the outputs of ac circuits. You will use such diagrams to model a circuit that has two or more sources of electricity.

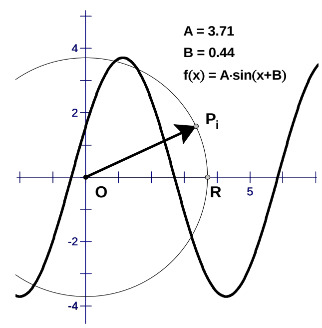
PROPERTIES OF PHASORS

In the steps that follow, you will explore the relationship between a sinusoid defined by $A \cdot \sin(x + B)$ and its corresponding phasor.

1. Open **Phasor.gsp** in the folder **Supplemental Activities | Phasor Diagrams**. The first page contains a coordinate system and a circle with its center at the origin. Move point R to change the radius of the circle.


To choose a custom tool, press and hold the **Custom** tools icon and choose the desired tool from the menu that appears.

2. Choose the **Initial Phasor** custom tool. Click on the circle to construct an initial phasor. The phasor appears, along with measurements of its magnitude and initial phase.



3. Label the endpoint of the initial phasor P_i by choosing the **Text** tool, clicking the point to show its label, then double-clicking the label to change it, and typing `P[i]`for the new label.
4. Measure $\angle ROP_i$ by choosing the **Radian Measure** custom tool and clicking in order on points R, O, and P_i.

Q1 What existing measurement corresponds to $m\angle ROP_i$? Why do they correspond? Are they always equal?

Q2 What existing measurement equals the x-value of point R? Why are they equal?

Q3 How can you calculate the x-value of point P_i from the values of A and B? Use Sketchpad's Calculator to do the calculation, and then measure the x-value of P_i so you can compare the results.

Q4 How can you calculate the y-value of point P_i from the values of A and B? Check your result.

So far you have constructed only the initial position of the phasor. The phasor itself starts at the initial position and rotates.

5. Construct the phasor by choosing the **Phasor** custom tool and clicking it on point P_i. The tool creates several additional objects, including an action button and a point labeled W.

6. To start up the phasor, choose the **Arrow** tool and click the *Start Phasor* button.

To trace point *W*, select it and choose **Display | Trace Plotted Point**.

Q5 As the phasor rotates, point W also moves. Trace this point and describe its motion.

7. Show the equation and graph of the sinusoid associated with this phasor by clicking the **Sinusoid** custom tool on measurement A (the phasor's magnitude) and then on measurement B (the initial phase).

Q6 How does the trace of point W relate to the sinusoid?

To turn off tracing, select point W and choose **Display | Trace Plotted Point** a second time. To erase traces, choose **Display | Erase Traces.**

8. Turn off tracing for point W, and erase the existing traces.

Q7 What happens when you change the value of A by dragging point R? What is the effect on the initial phasor? What is the effect on the sinusoid?

Q8 What happens when you change the value of B by dragging point P_i around the circle? What is the effect on the initial phasor? How is the sinusoid transformed?

Use 1.57 as an approximation for $\pi/2$ and 3.14 as an approximation for π..

Q9 Stop the rotation and drag point P by hand. What is position of the phasor relative to the initial phasor when $x = 0$? When $x = \pi/2$? When $x = \pi$? Confirm your answers for two different positions of P_i.

Measuring the coordinates of point *W* will help you answer this question.

Q10 Restart the rotation. As the phasor rotates, point W moves along the sine wave. To what feature of the phasor does point W's x-value correspond? To what feature of the phasor does point W's y-value correspond? Check your answers for two different positions of P_i.

Q11 What are the coordinates of point W when $x = 0$? When $x = \pi/2$? When $x = \pi$? Express your answers in terms of A and B, so that the are correct for different positions of P_i.

Q12 If you are given a sinusoidal curve, can you construct the corresponding initial phasor? To do so, how should you determine from the curve the two values you need—magnitude (A) and initial direction (B)?

PHASOR DIAGRAMS

Electrical circuits often contain several sources of ac current or voltage. To analyze the behavior of such circuits, engineers must calculate how the various currents or voltages combine. When the various sources have the same period but different magnitudes and phase shifts, phasors provide a convenient way to do these calculations. To add the sinusoidal curves representing the different sources, you can combine the phasors that correspond to each curve.

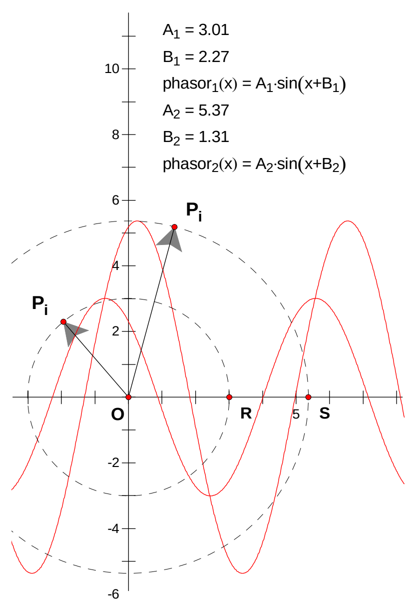
You have already seen that any sinusoidal function in the form of $A \cdot \sin(x + B)$ can be represented by a phasor with magnitude A and initial phase B. An expression involving the sum of two such functions can be represented as the sum of the corresponding phasors. Since phasors with the same period of rotation behave like vectors for any given value of x, you can apply the rules for adding vectors.

Combining Two Phasors

You can move points *R* and *S* to change the radii of the circles.

9. Page 2 of **Phasor.gsp** contains two circles that you can use to determine the magnitudes of two phasors. Use the **Phasor and Sinusoid** tool to construct a phasor on the inner circle. Change the labels of the measurements to A_1 and B_1, and change the label of the function to $phasor_1(x)$. Choose a color for this phasor and all its associated features (its measurements, its function, and its curve).

Q13 What are the coordinates of the endpoint of the initial phasor in terms of A_1 and B_1?



10. Use the **Phasor and Sinusoid** tool again to construct a second phasor, using the outer circle. Change the labels of its measurements and function to A_2, B_2, and $phasor_2(x)$. Choose a color for this phasor and its associated features to distinguish it from the first phasor.

Q14 What are the coordinates of the endpoint of this initial phasor in terms of A_2 and B_2?

11. Construct the vector sum of the two initial phasors by choosing the **Vector Sum** custom tool and clicking on the endpoint of each phasor.

12. Drag the appropriate points to change the values of A_1, B_1, A_2, and B_2. As you do so, observe the effect on the sinusoids and on the vector sum.

13. Use the **Phasor and Curve** tool to construct a phasor for the vector sum. Change the labels of its measurements and function to A_3, B_3, and $phasor_3(x)$.

Choose a different color for this phasor and all its features. Make its curve thick, to further distinguish the curve from the two original curves.

14. Once again, drag the points that determine A_1, B_1, A_2, and B_2, and observe the effect on the curve that corresponds to the vector sum.

The Combined Phasor in Terms of A_1, B_1, A_2, and B_2

Q15 Write a formula for the x-value of the endpoint of the vector sum in terms of A_1, B_1, A_2 and B_2.

Q16 Write a similar formula for the y-value of the endpoint of the vector sum.

Q17 Use these result to write an expression for A_3 in terms of A_1, B_1, A_2 and B_2. Then write an expression for B_3 in terms of A_1, B_1, A_2 and B_2. Use Sketchpad's Calculator to check your result, by calculating both expressions and comparing the results to the actual values of A_3 and B_3.

15. Substitute these two expressions into the formula $phasor_3(x) = A_3 \cdot \sin(x + B_3)$. The result is a function definition for $phasor_3(x)$ in terms of A_1, B_1, A_2, and B_2. Plot this function to confirm that it is identical to the function you have already plotted.

The function you have just plotted is the function of the sine wave that results in an electric circuit containing the two sources of electricity defined by phasors having magnitude A_1 and A_2 and phase shifts B_1 and B_2.

EXPLORE MORE

Present your work on page 2 by using the **Phasor** tool to construct rotating phasors for the three initial phasors (numbered 1. 2. and 3). Create a Presentation action button that starts all three phasors rotating at the same time.

Use phasors to model an electric circuit containing three sources of electricity A_1, B_1, A_2, B_2, A_3, and B_3. Use vector addition to add the three initial phasors, and to show the resulting sinusoid.

Use phasors to solve this equation geometrically: $3\sin(x+1) + 5\sin(x+2) = 6$. (Hint: construct a phasor for each of the two sine functions, and construct the combined phasor. For what values of x is the endpoint of the combined phasor on the line defined by $y = 6$?)

PHASOR DIAGRAMS

Objective: Students explore the relationship between phasors and their related sinusoids, and use phasors to combine two sinusoidal functions with the same period.

Prerequisites: Students should be familiar with magnitude and direction of vectors and with vector addition.

Sketchpad Proficiency: Intermediate. Students must be able to use custom tools.

Class Time: 40–50 minutes. If students are less experienced with The Geometer's Sketchpad, they can do the Properties of Phasors section in one session, and complete the remainder of the activity in a second session.

Required Sketch: Phasor.gsp

Presentation Sketch: Phasor Present.gsp

PROPERTIES OF PHASORS

Before starting the activity, you should review vectors and their properties. This will help them understand the differences between vectors and phasors, and finding the sum of two vectors is an important part of the activity.

Q1 Measurement B corresponds to $m\angle ROP_i$. They are equal in Quadrants I and II, but differ by 2π in Quadrants III and IV, because B is in the range from 0 to 2π, while $m\angle ROP_i$ is in the range from $-\pi$ to π.

Q2 Measurement A is always equal to the x-value of R. The phasor's endpoint is on the circle, so its magnitude is always equal to the radius of the circle.

Q3 To calculate the x-value of P_i, use $A \cos B$.

Q4 To calculate the y-value of P_i, use $A \sin B$.

Q5 Point W moves to the right while its vertical position matches the y-value of P_i. It traces out one cycle of a sinusoid.

Q6 The trace of point W exactly matches the sinusoid from 0 to 2π.

Q7 When you drag point R to change the radius of the circle (and the value of A), the initial phasor gets longer or shorter to match. The sinusoid's amplitude changes to match. The initial phasor's direction, the period of the sinusoid, and the phase shift of the sinusoid all remain unchanged.

Q8 When you drag point P_i to change the value of B, the direction of the initial phasor changes, and the phase shift of the sinusoid changes. The magnitude of the phasor and the amplitude of the sinusoid remain unchanged.

Q9 When $x=0$, the phasor and the initial phasor are coincident. When $x=\pi/2$, the phasor is perpendicular to the initial phasor. When $x=\pi$, the phasor is opposite to the initial phasor.

Q10 The x-value of point W corresponds to the angle of the phasor relative to the initial phasor. The y-value of point W corresponds to the y-value of the the phasor's endpoint.

Q11 When $x=0$, point W is at $(0, A \sin B)$. When $x=\pi/2$, point W is at $(\pi/2, A \sin [B+\pi/2])$. When $x=\pi$, point W is at $(\pi, A \sin [B+\pi])$, which can also be expressed as $(\pi, -A \sin B)$.

Q12 Given a sinusoidal curve, you can construct the initial phasor by setting its initial direction (B) to the negative of the rising x-intercept and by setting its magnitude (A) equal to the amplitude of the sinusoid.

PHASOR DIAGRAMS

Q13 The endpoint of the first initial phasor is $(A_1 \cos B_1, A_1 \sin B_1)$.

Q14 The endpoint of the second initial phasor is $(A_2 \cos B_2, A_2 \sin B_2)$.

Q15 The x-value of the endpoint of the vector sum is $A_1 \cos B_1 + A_2 \cos B_2$.

Q16 The y-value of the endpoint of the vector sum is $A_1 \sin B_1 + A_2 \sin B_2$.

Q17 Using the Pythagorean Theorem for A_3,

$$A_3 = \sqrt{(A_1 \cos B_1 + A_2 \cos B_2)^2 + (A_1 \sin B_1 + A_2 \sin B_2)^2}$$

To calculate B_3, take advantage of the fact that the tangent of the initial angle is equal to y/x:

$$B = \text{Arc tan}\left(\frac{A_1 \sin B_1 + A_2 \sin B_2}{A_1 \cos B_1 + A_2 \cos B_2}\right)$$

EXPLORE MORE

The first question involves using the Phasor tool three times, on each of the three initial phasor endpoints, and then creating a Presentation button to activate the three *Rotate Phasor* buttons simultaneously.

The second question, with three sources of electricity, is an extension of what students have already done with two sources, and requires them to find the vector sum of three initial phasors.

Here's one way to solve the equation in the third question. Construct initial phasors for the two functions and find the phasor for the combined function. An easy way to construct the initial phasors is to first plot the two points below, in polar coordinates, and then click the **Initial Phasor** tool on each of the plotted points.

Point	Corresponding Expression
(3, 1)	$3 \sin (x+1)$
(5, 2)	$5 \sin (x+2)$

Once you've constructed the two initial phasors, construct the combined phasor. Then construct the line $y = 6$ by plotting the point $(6, \pi/2)$ and constructing a perpendicular to the y-axis through this point.

The solutions can be found from the intersection points of the circle determined by the combined phasor with the horizontal line $y = 6$.