When you're riding in a car, it appears that other objects in the car remain fixed in position, while the buildings and scenery outside appear to be in motion. Similarly, when you're on a carousel, it appears that the world around you is spinning.

In both these examples, your perceptions are based on your frame of reference: you experience the buildings and scenery from your own point of view, inside the moving car or on the carousel. Someone standing on the sidewalk has a very different frame of reference. A pedestrian on the sidewalk might claim that you (in the moving car) have a moving frame of reference while she is standing still. But even the pedestrian's frame of reference is moving, because the surface of the earth is rotating at a speed of about 1000 mph, while the earth itself is rotating about the sun, and the sun is rocketing through space.

There are many problems in physics and mathematics that you can solve more easily by adopting a particular frame of reference. In this activity you will explore several such problems by using Sketchpad to create a frame of reference that simplifies the problem at hand.

LINEAR MOTION

Consider an elementary word problem:

Car B is 1 km east of car A. Car A is traveling east at a speed of 80 km/h. Car B is traveling west at 40 km/h. How long will it take for the two cars to meet?

Although this is not a difficult problem, it is a good one to illustrate how changing the frame of reference can simplify the solution.

1. Open **Frame of Reference.gsp** in the folder **Supplemental Activities | Frame of Reference**. The first page models this problem using a Sketchpad animation. When you click the *Animate Cars* button, the two cars move toward each other, with car *A* moving twice as fast as car *B*.

You could solve this problem by creating a system of equations, with one equation representing the motion of each car. However, you can solve it with a single equation by looking at it in a different way.

To mark vector *AA'*, select the two points in order and choose **Transform | Mark Vector**. The translate the road and cars, select them and choose **Transform | Translate**.

2. Below the road is a point labeled *A'*. Translate the road and both cars by vector *AA'*. Reset the cars and click the *Animate Cars* button again.

Q1 The translated images in the bottom part of the drawing are in a different frame of reference. What can you say about the motion of car *A* in the transformed frame of reference? What can you say about the motion of car *B*?

©2005 Key Curriculum Press

Clearly, the time elapsed must be the same in both frames of reference. Relative to the position of car A, the speed of car B is 120 km/h. You have simplified the problem by viewing it as a single car traveling 1 km at a speed of 120 km/h.

CIRCULAR MOTION

At 12 o'clock, the hour hand and minute hand of a clock are pointing in the same direction. At what time (to the nearest second) will the next alignment occur? You can solve this problem more easily by changing the frame of reference.

3. Open the Clock page of **Frame of Reference.gsp**. The *Animate Hands* button causes the hands to go in motion with the correct relative speed, albeit faster than a normal clock.

A helpful way to approach this problem is to imagine the hour hand being stationary and consider the angular speed of the minute hand with respect to it. To accomplish this in Sketchpad, you must rotate the entire clock by an angle that will keep the hour hand pointing straight up.

Q2 Stop the animation in a position in which the hour hand is not pointing straight up. By what angle must you rotate the clock to make the hour hand point straight up?

If the rotated hour hand doesn't point straight up, think again about the angle by which you must rotate.

4. Select, in order, the three points that determine this angle. Choose **Transform | Mark Angle**. Mark center point O. Rotate the clock and both hands by the marked angle.
5. Click the *Hide Pre-image* button, so you can see only the rotated clock.

When you press the *Animate Hands* button again, the hour hand is stationary, but the minute hand and the clock face rotate at the appropriate speeds relative to the hour hand. Now you only need to figure out how long it takes the transformed minute hand to make one complete revolution.

Q3 In the normal frame of reference, the hour hand rotates at an angular speed of 2 cycles per day. What is the angular speed of the minute hand?

Hint: you can count the number of revolutions of the transformed minute hand in 24 hours.

Q4 In the rotated frame of reference, the hour hand does not rotate at all. What is the angular speed of the minute hand in this frame of reference?

Q5 How long does it take for the transformed minute hand to make one revolution?

Q6 What time is the next alignment after 12 o'clock?

©2005 Key Curriculum Press

ASTRONOMY

The word *planet* comes from a Greek word meaning "wanderer."

Early astronomers observed that stars held stable relative positions as they progressed across the sky. However, there were other points of light that refused to stay in step. These were the planets. To the unaided eye, there is little visible difference between a star and a planet, but they behave very differently over time.

The backward loop Mars makes relative to the stars is called *retrograde motion*.

Early astronomers observed strange behavior in the motion of Mars. It not only changed speed, but also direction, periodically spinning a loop in the sky. Early astronomers did not realize that the motion of Mars was really quite regular, and that it looked erratic because it was being viewed from a moving platform—the Earth.

6. Open the Planets page of **Frame of Reference.gsp**. This page contains a rough model of the orbits of Earth and Mars, using the sun to define the frame of reference. Press the *Animate Planets* button to watch the motion of the planets.

In the real solar system, the orbits of Mars and Earth are not quite circular, but are slightly elliptical in shape.

Viewed in the sun's frame of reference, the orbit of Mars is a circle. Earth has a smaller orbit and moves faster, which is why we lap Mars about every 2.1 years.

7. Investigate the motion of Mars as viewed from Earth. To make Earth stationary, use the stationary point *Earth'* on the right of the screen, and translate the entire solar system by the vector from *Earth* to *Earth'*. Animate the planets again.

Now the frame of reference is the Earth—the same as experienced by star-gazers, ancient and modern. Earth appears stable while Mars and the sun revolve around it.

Q7 Notice the irregular path of Mars. Trace its motion, and describe its path relative to Earth. What does the retrograde motion look like, and why does it occur?

EXPLORE MORE

8. Construct a frame of reference to show what the orbit of Earth looks like from Mars.

9. Choose **Display | Show All Hidden** to see the construction of the orbits. The position of Earth is a rotation based on the parameter theta. For Mars, theta is divided by 1.9, because a year on Mars is 1.9 times as long as a year on Earth. Use the following information on Venus and Jupiter to construct their orbits. You will need to change the scale in order to fit Jupiter's orbit into the picture.

	Semimajor Axis (AU)	**Sidereal Period (years)**
Venus	0.7	0.6
Jupiter	5.2	11.9

©2005 Key Curriculum Press

FRAME OF REFERENCE

Objective: Students modify the frame of reference in order to simplify several problems: a moving-car problem, a clock-hand problem, and a retrograde-motion problem. They solve the problems using moving and rotating frames of reference.

Prerequisites: None.

Sketchpad Proficiency: Intermediate. Students will find the activity easier if they are already familiar with translation by a marked vector and rotation by a marked angle.

Class Time: 30–40 minutes

Required Sketch: Frame of Reference.gsp

LINEAR MOTION

Q1 In the transformed frame of reference, car A is stationary, and car B moves more quickly. The relative speed of the two cars remains the same.

CIRCULAR MOTION

Q2 To make the hour hand point straight up, you must rotate the clock by $\angle HOXII$.

Q3 The minute hand rotates at a speed of 24 cycles per day, or 1 cycle per hour.

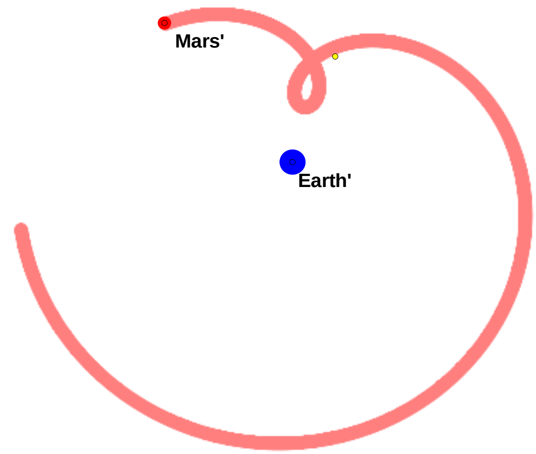
Q4 In the rotated frame of reference, the minute hand rotates at 22 cycles per day, or 11 cycles every 12 hours.

Q5 The transformed minute hand makes a full revolution every 12/11 hour, or every 65.4545... minutes. This can also be expressed as 1 hour and 5.4545... minutes, or 1 hour, 5 minutes, and 27.2727... seconds.

Q6 The next alignment will be at 1:05:27.2727... o'clock.

ASTRONOMY

Q7 The retrograde motion of Mars involves a loop in which it comes closer to Earth and moves in the opposite direction from normal:



The retrograde motion occurs as Earth catches up to and passes Mars. The Earth's greater speed makes it look as though Mars is moving backward relative to the stars, even though the motion of Mars in the sun's frame of reference remains unchanged.

©2005 Key Curriculum Press