

This document describes all the activities in *Exploring Precalculus with The Geometer's Sketchpad*.

The Proficiency column (describing the prior experience students should have before attempting the activity) uses the following terms:

**Beginner:** Students need have no prior Sketchpad experience. The activity either involves manipulation of a prepared sketch, or it contains detailed instructions for all required constructions and measurements.

**Intermediate:** Students should have some experience with simple constructions and measurements, with changing parameter values, or with plotting functions. Hints are usually included (in the form of margin notes) for many of these operations.

**Advanced:** Students should be comfortable with a variety of Sketchpad operations. The instructions in the activity are at a higher level of abstraction, describing the desired result of a construction rather than specifying each selection to be made and each command to be used.

### **Chapter 1: Function Transformations**

<b>Page</b>	<b>Title</b>	<b>Description</b>	<b>Proficiency</b>	<b>Time</b>
3	Translation of Functions	Students create both geometric and algebraic translations of a function, match the two translations, and use the components of the geometric vector to write an equation for a translated image.	Beginner	25-30
8	Dilation of Functions	Students explore dilations of functions geometrically and algebraically and learn to write equations for dilated functions.	Beginner to Intermediate	30-40
12	Reflection of Functions	Students explore reflections across the x- and y-axes algebraically and geometrically and write equations for reflected functions.	Intermediate	20-30
14	Absolute Value of Functions	Students explore absolute value transformations and investigate the connection between absolute value and reflection.	Intermediate	20-25
16	Composition of Functions	Students use dynagraphs to analyze composition of functions, domain and range of composite functions, and commutativity of function composition.	Beginner to Intermediate	30-40
19	Inverses of Functions	Students explore inverses of functions graphically and symbolically and determine when inverse functions exist.	Intermediate	30-35
22	Transformation Challenge	Students challenge each other with transformed functions, looking at the graph representation and trying to deduce the algebraic representation.	Beginner	15-20

## **Chapter 2: Circular Functions**

25	Introduction to Radians	Students learn about radian measure and investigate a Sketchpad simulation that approximates $\pi$ to two decimal places.	Beginner	20–30
27	Trigonometry Tracers	Students construct a unit circle and use a point on the circle to define and plot circular functions.	Beginner	20-30
29	Six Circular Functions	Students create a diagram containing six segments representing the six circular functions and use it to analyze the behavior of the six functions. The diagram can serve as a convenient mnemonic device to help students remember how the functions behave.	Intermediate	35-40
31	Transformations of Circular Functions	Students use a point on the unit circle to define and plot transformed circular functions.	Beginner	20-30
32	Sine Challenge	Students are challenged to create a sine function so that the first crest of the graph passes through a given point.	Advanced	20-30

## **Chapter 3: Trigonometric Properties**

35	The Law of Sines	In this activity, students begin by computing sine ratios in two right triangles. They then use Sketchpad to merge the triangles. By studying the resulting triangle, students derive the Law of Sines.	Beginner	40-50
36	Law of Cosines	Students build a Sketchpad model that yields the formula for the Law of Cosines. The proof relates $c^2$ , $a^2$ , and $b^2$ to the areas of three squares and $2ab \cos(\theta)$ to the combined areas of two congruent parallelograms.	Intermediate	40-50
38	Sums of Sinusoidal Functions	Students model wave superposition and wave packets, analyzing sums of sinusoidal waves.	Intermediate to Advanced	40-50
42	Products of Sinusoidal Functions	Students graph products of sinusoidal equations and study their properties. This is a continuation of the Sinusoidal Sums activity.	Intermediate	40-50

### Chapter 4: Other Functions

49	Exploring the Roots of Quadratics	Students use linked coordinate systems to explore the relationship between the roots of a quadratic function and the shape of its graph, and to explore the relationship between the roots of a quadratic function and its coefficients. They then connect their results to the quadratic formula.	Beginner	40-50
51	Analytic Conics	Students build their understanding of analytic representations of conics by making changes to two equation forms and immediately seeing changes in the resulting curve.	Beginner	30-40
53	Parametric Functions	Students explore parametric functions in rectangular and polar coordinates	Beginner	40-50
56	Surfaces	Students manipulate prepared sketches to view and observe the properties of quadric surfaces, including conic surfaces of revolution and a hyperbolic paraboloid.	Beginner	30-40
59	Compound Interest	Students use iterated calculations to compute and plot the value of a compound interest investment. This value is connected to the constant $e$ and the general formula for continuous compounding.	Intermediate	30-40
62	Slopes of Exponential Functions	By observing graphs, students make inferences about the relationship between exponential functions and their slopes.	Intermediate	30
64	A Sequence Approach to Logs	Students graph geometric sequences against arithmetic sequences to obtain good approximations of log curves.	Beginner	40-50
66	Semilog Graphs	Students explore a logarithmic scale and a semilog grid, and use the grid to investigate exponential functions and to solve a problem involving exponential decay.	Intermediate	50-60
70	Log-Log Graphs	Students explore graphs of power functions on a log-log grid, and use the grid to solve an interesting statistical problem.	Intermediate	40-50
73	The Logistic Function	Students build a Sketchpad model of a population for which the growth is restrained by some factor. In the process, students explore the sensitivity of the long-term behavior to the initial size of the population and to the parameters that determine the population growth.	Intermediate	40-50

### **Chapter 5: Data and Probability**

79	Linear Regression	Students manipulate scatter plots and experiment with best-fit lines and acquire an intuitive feel for regression lines, residuals, and least squares analysis.	Intermediate	30-40
81	Wait for a Date	Students use a pre-made Sketchpad model to gather sample data for a probability problem. By viewing the data as points in a plane, students uncover a geometric pattern that allows them to compute a precise probability.	Beginner	40-50
83	Fitting Polynomial Functions	Students will need to be familiar with the concepts of matrix multiplication and the inverse matrix. Custom tools perform the calculations, so it is not necessary for the student to actually process all of the numbers.	Intermediate	30

### **Chapter 6: Vectors and Matrices**

87	Matrix Transformations	Students perform elementary coordinate transformations using matrices and vector addition.	Intermediate	30-40
89	Vector Operations	Students observe geometric interpretations of a dot product and a vector projection, and then compute and plot a vector projection.	Intermediate	30-40
92	Matrix Products	Students create complex transformations using the product of several transformation matrices.	Intermediate	50-60
95	Coordinates in Three Dimensions	Students explore coordinates in three dimensions using rectangular, cylindrical, and spherical coordinate systems.	Intermediate	30-40
98	Parametric Functions in Three Dimensions	Students use parametric functions to draw curves in rectangular $(x, y, z)$ coordinates. An open-ended extension uses cylindrical and spherical coordinates.	Beginner	30-50
101	Vector Operations in Three Dimensions	Students use custom tools to construct the cross product of three-dimensional vectors and use vector operations to determine collinearity and coplanarity of points.	Advanced	40-50
103	Matrix Transformation in Three Dimensions	Students use matrices to transform a shape in three dimensions.	Intermediate	40-50
106	Matrix Products in Three Dimensions	Students use three-dimensional transformation matrices to simulate the motion of an orbiting planet. This is a three-dimensional extension of the two-dimensional Matrix Product activity.	Beginner to Intermediate	30-40

## Chapter 7: Polar Coordinates and Complex Numbers

111	Introduction to Polar Coordinates	Students experiment with $(r, \theta)$ values in polar coordinates by adjusting parameters and find multiple ways of representing the same point. Students then use a right triangle to develop formulas for $x$ and $y$ in terms of $r$ and $\theta$ .	Intermediate	25-35
113	Cartesian Graphs and Polar Graphs	Students compare rectangular graphs and polar graphs for functions in the form $y = a \sin(bx)$ and $r = a \sin(b\theta)$ . Students analyze how the period and amplitude of a Cartesian graph correlate with features of the corresponding polar graph.	Beginner	20-30
115	Multiplication of Complex Numbers	Students use a set of Sketchpad custom tools to build, step by step, the product of two complex numbers. By analyzing their construction, students develop a geometric understanding of complex number multiplication.	Beginner	40-50
118	In Search of Buried Treasure	Students construct a sketch that models the directions given to unearth a buried treasure. They make a conjecture about the treasure's location and then prove their results by overlaying the complex plane onto their map.	Intermediate	40-50
120	Transformations in the Complex Plane	Students explore a collection of sketches, each of which displays a complex number, $z$ , and, simultaneously, another complex number, $w$ . The mystery number $w$ is computed as a function of $z$ . By dragging $z$ and watching the behavior of $w$ , students match each $w$ to an equation listed in the activity.	Beginner	30-40
121	Power of Complex Numbers	By exploring several interactive models, students discover that multiple values of $z$ can satisfy an equation like $z^3 = 2i$ . As students explore higher and higher powers of $z$ , they begin to see patterns that form the basis of De Moivre's Theorem.	Beginner	40-50
123	A Geometric Approach to $e^{i\pi}$	Students use a limit definition of $e$ , along with multiplication on the complex plane, to find the value of $e^{i\pi}$ .	Beginner	40-50

## **Chapter 8: Sequences and Series**

129	Generating Arithmetic and Geometric Sequences Numerically	Students develop an understanding of arithmetic and geometric sequences by building and modifying them with Sketchpad.	Beginner	25-35
131	Area Models of Geometric Series	Students use dissections of a square to represent geometric series and to investigate sums of these series.	Advanced	40-50
134	A Geometric Series Coil	Through a visual and numerical exploration, students see firsthand what happens to a geometric series when the number of its terms grows larger and larger.	Beginner or Intermediate	40-50
137	A Geometric Series Staircase	Students build a model depicting a staircase of shrinking squares. They examine the model to find a segment whose length represents the sum of an infinite geometric series. Through the application of similar triangles, students derive the generalized algebraic formula for the sum of an infinite geometric series.	Intermediate	40-50
139	Taylor Series	Students create a Taylor series and explore how adding terms to a Taylor series approximation increases the accuracy of the approximation.	Intermediate	30-40

## Chapter 9: Introduction to Calculus

145	Instantaneous Rate	Students make a connection between instantaneous rate and slope of the tangent to the graph, see the instantaneous rate as a limit of the slope between two points (just as the tangent represents the limit of a secant line), and are introduced to the concept and definition of the derivative.	Beginner	30-40
147	One Type of Integral	Students explore the concept of definite integral—the accumulation of a function’s values over a particular domain of the independent variable. The practical application, in which students use the velocity function to determine distance traveled, helps to suggest the usefulness of the definite integral. This activity emphasizes the nature of the definite integral as the accumulation of the value of a function over a particular domain. Counting squares makes the “accumulation of value” concrete and can also leave students motivated to learn the mathematics that will enable other less tedious methods.	Beginner	40-50
149	Rectangular and Trapezoidal Accumulation	Students develop a way of accumulating values that is more accurate and more efficient than counting squares.	Intermediate	25-30
151	Limits with Tables	Students explore the fundamental concept of a limit numerically, by looking at values in a table.	Beginner	30-40
153	Limits with Delta and Epsilon	Students explore the formal definition of a limit and by choosing values of $L$ (to determine the limit itself) and manipulating values of $\delta$ and $\epsilon$ (so that the definition actually makes sense). By manipulating the sliders for $\delta$ and $\epsilon$ , students get a much clearer sense of the meanings of these two values and a clearer understanding of the formal definition of a limit.	Beginner	30-40
155	Manually Probing the Antiderivative	Students use what they know about derivatives and slopes to trace an antiderivative.	Intermediate	20-30
157	Automatically Probing the Antiderivative	Students continue the previous activity by automating the tracing process, using a movement button and using the Iterate command.	Intermediate	20-30

## Supplemental Activities

	Spirals	Students learn properties of three kinds of spirals, presented as continuous curves and as discrete objects on those curves.	Beginner	40
	Cycloid	Students use vector addition to define parametric functions for a cycloid.	Intermediate	30-40
	Epicycloid and Hypocycloid	Students apply the concept of vector addition to define parametric functions for epicycloids and hypocycloids.	Intermediate	40-50
	Phasor Diagrams	Students explore the relationship between phasors and their related sinusoids, and use phasors to combine two sinusoidal functions with the same period.	Intermediate	40-50
	Frame of Reference	Students modify the frame of reference in order to simplify several problems: a moving-car problem, a clock-hand problem, and a retrograde-motion problem. They solve the problems using moving and rotating frames of reference.	Intermediate	30-40
	Chaos (Part 1)	The two parts of this activity can be done separately. In part 1 students create a Sierpiński triangle fractal design using strange attractors.	Intermediate	30-40
	Chaos (Part 2)	The two parts of this activity can be done separately. In part 2 students create and explore Mira, Julia, and Mandelbrot fractals.	Intermediate	30-40
	The Mandelbrot Set	Students investigate the Julia mapping and see how it is used to generate the Mandelbrot set. The accompanying sketch produces a beautiful colored image of the Mandelbrot set and allows the student to zoom in on any portion of the set.	Intermediate	40
	Barnsley's Fern	Students plot points using four pairs of iterated functions. By choosing randomly among the function pairs, they create a fractal Barnsley's Fern.	Advanced	40

## Supplemental Sketches

Parametric Linkage.gsp	Students operate a model that uses linkages to produce Lissajous figures, and adjust the linkages to change the amplitudes, periods, and phase shifts of the two equations.		
Piano.gsp	Students press the piano keys to create chords or dissonances, and see the shapes of the resulting sound waves.		
3D Graph Template 1.gsp	Students can plot points and curves conveniently in three dimensions and view the three-dimensional creation from different angles. The sketch includes graphing tools.		
3D Graph Template 2.gsp	Students can plot in three dimensions and change the viewing angles. This template provides sophisticated options (such as a perspective view), and is used in the 3D activities in this book. The sketch includes graphing tools.		
Trig Coords.gsp	Students can use the coordinate system in this sketch to plot functions using radians, expressed as fractions of $\pi$ , on the $x$ -axis.		
Zooming Coordinate System.gsp	Students can use this coordinate system to plot functions and then zoom in or out on the resulting graphs.		
Light Speed.gsp	This sketch illustrates a wave packet traveling faster than the speed of light, and shows how such a packet cannot overcome the limitations of Einstein's Theory of Special Relativity.		
Graph Paper.gsp	Students and teachers can design their own grids and graph on them. The grid is limited to user-defined region of the sketch. Students and teachers can prepare special grids for printing or for pasting into worksheets or other documents.		

## **Supplemental Tools**

	0-2pi Radian Measure.gsp	These tools allow students to measure angles in radians, expressed as fractions of $\pi$ or as decimal values between 0 and $2\pi$ .		
	3D Tools.gsp	Students use these tools to create 3D coordinate systems and plot points on them using rectangular, cylindrical, or spherical coordinates.		
	Conics Plus.gsp	Students use these tools to construct geometric loci of parabolas, ellipses, and hyperbolas using a variety of methods.		
	Conic Intersections.gsp	Students use these tools to find the intersections of straight objects with parabolas, ellipses, or hyperbolas defined as loci.		
	Intersect Functions.gsp	These tools find the first zero of a function, or the first intersection of two functions, within a specified domain.		
	Matrix Brackets.gsp	This appearance tool creates adjustable matrix brackets that can be sized and positioned to enclose a rectangular array of parameters.		
	Matrix Tools.gsp	Students can multiply a matrix by a vector or another matrix, and find the determinant, adjoint, or inverse of a matrix. The sketch contains tools for 2-, 3-, 4-, and 5-dimensional matrices.		
	Vector Tools.gsp	Students can find vector sums, lengths, and dot products in 2, 3, 4, and 5 dimensions, and vector cross products in 3 dimensions.		