Function Transformation Game

By now you have read all about function transformations. That was theory; now comes practice. For a quick review, here is the equation for the graph of a parent function $p(x)$ transformed four ways:

$$y = a \cdot p\left(\frac{x - h}{b}\right) + k$$

Here is the transformation determined by each of the four parameters:

a: vertical stretch

b: horizontal stretch

h: horizontal translation

k: vertical translation

Open **Function Transformation Game.gsp.** There are two graphs on the screen. If this is your first time playing the game, disregard the red graph for now while you get used to the controls for the blue graph.

The blue graph, $y = g(x)$, is a transformation of the parent function highlighted in yellow. Press *Choose Parent Function* to step through the six available parent functions. Parameters *a, b, h,* and *k* define the transformation. Select a parameter and press the + or − key to change it.

The red graph, $y = f(x)$, is your objective. You must make the graph of $g(x)$ fit right on top of it. When you think you have the correct solution, press *Show Results.* A message tells you whether you have succeeded, and the definition of $f(x)$ appears. Don't be surprised if the function definitions do not match exactly even when the graphs match. It is often possible to define a given transformation in more than one way. Press *Play* to create a new random definition for $f(x)$.

It may be difficult to get precise coordinates from looking at the graph. You can use parameter t to help you. Enter any number for t to see the value of $f(t)$, even if the corresponding point is off the screen.

Once you really, really think you know what you are doing, press *Hide Graph of* $g(x)$. Now you are flying on instruments. This is much more difficult. You still have complete control of $g(x)$, but you cannot see its graph until you show the results.

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Activity Notes

Objective: Students match the graph of a mystery function by choosing a parent function and applying transformations to it.

Student Audience: Algebra 2/Precalculus

Prerequisites: Students should be familiar with all four function transformations used here: horizontal stretch, vertical stretch, horizontal translation, and vertical translation. They should also know the parent functions: x^2, $\sqrt{x}$, 2^x, x^{-1}, $\sin x$, and $\log x$.

Sketchpad Level: Easy. Students manipulate a pre-made sketch.

Activity Time: Flexible

Setting: Paired/Individual Activity (use **Function Transformation Game.gsp**)

PLAYING THE GAME

You may want to briefly show the sketch to familiarize students with the mechanics and objective of the game, or you may want students to jump in immediately and experiment on their own. If you show the sketch first, be quick to get students working on their own. Students are not nearly as engaged watching someone else transform functions as they are doing it themselves.

Encourage students to work in pairs and to discuss their strategies.

Some of the most interesting results occur when the graphs match but the function definitions do not. Have students record the equivalent functions. Later you can help them explain the equivalence. Here are examples of identities that might occur:

$$4x^2 = \left(\frac{x}{0.5}\right)^2$$

$$3x^{-1} = \left(\frac{x}{3}\right)^{-1}$$

$$6 \cdot 2^{x-5} = 3 \cdot 2^{x-4}$$

$$-\sin x = \sin\left(\frac{x}{-1}\right)$$

RANDOMIZATION

Press *Play* to create a new definition for function $f(x)$. The button chooses a new parent function and a new set of transformations.

The *Play* button chooses the new parent function randomly from the six available parent functions. Each parent function has an equal probability of being chosen.

The *Play* button also chooses new transformations by assigning random values to each of the four parameters. The parameters are assigned randomly and independently, but in such a way that each parameter has a 0.5 probability of causing no transformation (a zero translation or a stretch factor of one). This way $f(x)$ is usually a combination of no more than two transformations; occasionally it will be more complex and occasionally less.

Translation parameters h and k will always be integers, and stretching parameters a and b will always be integer multiples of 0.5.