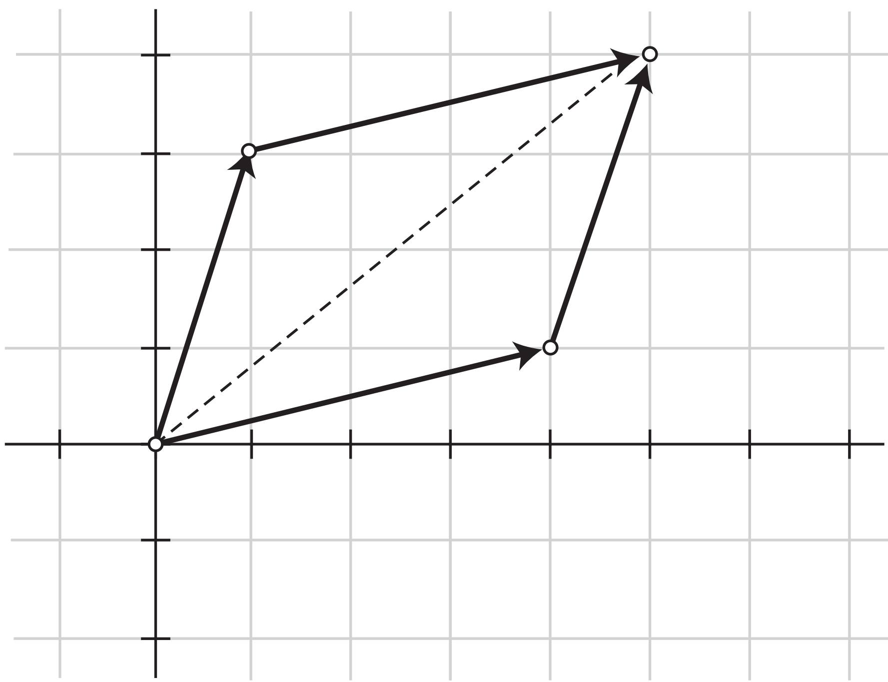
9

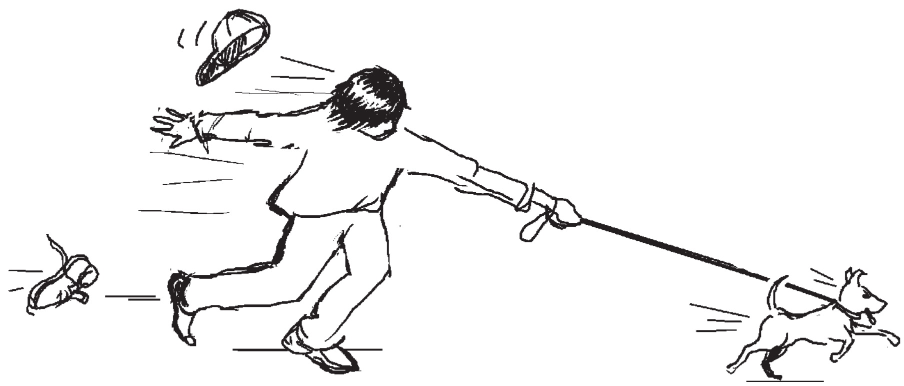
Vectors and Matrices



Introduction to Vectors: Walking Rex

You know that $2 + 2 = 4$. But if you walk 2 miles north and then 2 miles south, how far did you go? In one sense you went 4 miles—that's what your feet would tell you. But in another sense you haven't gone anywhere. We could say: $2N + 2S = 0$.

Values with both magnitude (size or length) and direction are called *vectors*. Vectors are useful in studying the flight paths of airplanes in wind currents and the push and pull of gravitational or electric forces. In this activity you'll explore the algebra and geometry of vectors in the context of a walk with your faithful dog, Rex.



WALK THE DOG

1. Open **Introduction to Vectors.gsp.** Rex's leash is tied to a tree at the origin of an *xy*-coordinate system. Rex is pulling the leash tight as he excitedly waits for you to take him on a walk.

Vectors often have a physical meaning. This particular vector represents Rex's position relative to the tree.

Rex's taut leash is represented by a vector, a segment with an arrowhead. The end with the arrow (Rex) is called the *head,* and the plain end is called the *tail.* We've labeled this particular vector **a**.

Q1 One way to define a vector is by its magnitude and direction. Which of these two quantities stays the same as you drag point *Rex*?

Another way to define a vector is by the coordinates of its head when its tail is at the origin. These coordinates are called the *components* of the vector.

You will not be able to make the components match the given values exactly. Just make them as close to those values as you can.

Q2 For each problem, drag *Rex* so the vector has the given components, and find the magnitude and direction of the vector.

a. components are $(5, 0)$

b. components are $(-4, 3)$

c. components are $(0, -5)$

d. components are $(-3, -4)$

Q3 For each problem, drag the vector as close as you can to the given magnitude and direction, and find the vector's components.

a. magnitude = 5; direction = 30°

b. magnitude = 5; direction = 135°

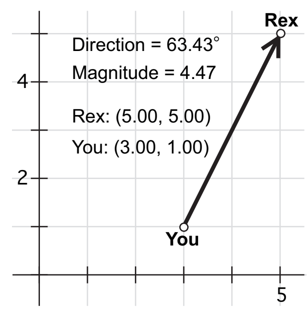
c. magnitude = 5; direction = 240°

d. magnitude = 5; direction = 307°

Q4 Rex is terrified of ladybugs. Suppose a ladybug is sitting at (5, 0). Where should Rex move to face in the opposite direction and be as far from it as possible? Describe Rex's position both ways, using components and using magnitude and direction.

Q5 What if the ladybug moves to a position 5 units away from the tree at 140°? Where should Rex go now?

Now it's time to untie the leash from the tree and take Rex for a walk.



2. Go to page 2. Rex is very determined! As you walk him, he pulls the leash taut and always tries to pull you in the same direction. Rex is still at the head of vector **a**, and now you're at the tail.

Q6 What are the components of vector **a**? How can you determine the components without moving point *You* to the origin?

Q7 Drag vector **a** around the screen. Explain why, no matter where you drag it, vector **a** is always the same vector. Support your argument using both of the two methods for describing vectors.

Q8 Suppose you walk to the point (80, 80). Where will Rex be? Explain how you found your answer. (Don't scroll—all the information you need is on the screen.)

3. Go to pages 3 and 4. Rex is heading in different directions on these pages. The information presented on screen is also a little different for each page.

Q9 On each of these two pages, determine where Rex will be standing when you're at (80, 80). Explain your reasoning in each case.

Q10 What if your leash is twice as long, and Rex is still pulling in the same direction? Now where will Rex be when you're at (80, 80)? Answer for both page 3 and page 4.

Introduction to Vectors: Walking Rex

Objective: Students learn two ways to describe vectors, convert between the two descriptions, and move vectors around to explore how the vector is independent of any specific position.

Student Audience: Algebra 1/Algebra 2

Prerequisites: None. This activity is designed to be a first introduction to vectors.

Sketchpad Level: Easy. Students work with a pre-made sketch.

Activity Time: 20–30 minutes. It may be possible to do this activity and the follow-up activity, Vector Addition and Subtraction, in one class period.

Setting: Paired/Individual Activity (use **Introduction to Vectors.gsp**) or Whole-Class Presentation (use **Introduction to Vectors Present.gsp**)

Related Activity: Vector Addition and Subtraction

WALK THE DOG

Q1 The magnitude stays the same. (In other words Rex is always the same distance—the length of his leash—from the tree.)

Q2 Students should be able to get within about 1° of the answers below.

a. magnitude = 5; direction = 0°

b. magnitude = 5; direction = 143.13°

c. magnitude = 5; direction = 270°

d. magnitude = 5; direction = 233.13°

Q3 Students should be able to get within about 0.1 of the answers below.

a. (4.33, 2.50)

b. (−3.54, 3.54)

c. (−2.50, −4.33)

d. (3.00, −4.00)

Q4 To get away from the ladybug when it's at (5, 0), Rex should move to (−5, 0). In magnitude-direction form he should move 5 units in direction 180°. In general, vectors (x, y) and $(-x, -y)$ face in opposite directions.

Q5 To get away from the ladybug when it's at distance 5 units and direction 140°, he should move to distance 5 units and direction 320°. In component form he should move to (3.83, −3.21). Assuming positive magnitude, a vector in direction θ and a vector in direction $\theta + 180°$ face in opposite directions.

Q6 The components of **a** are (2, 4). To find these components without moving point *You* to the origin, you can subtract the coordinates of the tail from the coordinates of the head.

Q7 The only things that change when vector **a** is dragged are the locations of its head and tail. The first method for describing vectors uses magnitude and direction—neither of these changes as **a** is dragged. The second uses the components, and these don't change either. No matter where the head and tail actually are, the result of the subtraction never changes.

Q8 (82, 84). No matter where you're standing, Rex is 2 units to the right and 4 units up, because the components of the vector are (2, 4).

Q9 Page 3: (83, 81). Find the components by dragging the vector so its tail is at the origin. The components are (3, 1), so Rex is always 3 units to the right of you and 1 unit up from you.

Page 4: (74, 83). Find the components by dragging the vector so its tail is at the point marked (8, 6) and then subtracting. The components are (−6, 3), so if you are at (80, 80), Rex will be at (74, 83).

Q10 Page 3: (86, 82). If the leash is twice as long, the components must be (6, 2).

Page 4: (68, 86). If the leash is twice as long, the components must be (−12, 6).

Use this presentation to introduce vectors and two common ways to describe them. Vectors often represent physical quantities, and the scenario of walking a dog on a leash provides a hook students can use to understand the concepts better.

Q1 If you walk 2 miles north and then 2 miles south, how far did you go? (Encourage discussion of the alternative answers: 4 miles or no distance at all. Use the discussion to point out that distance alone isn't enough here, and that you also have to take into account the direction of the walk.)

Define a *vector* as a quantity that has both magnitude (size or length) and direction, and describe some areas in which vectors are important in everyday life.

WALK THE DOG

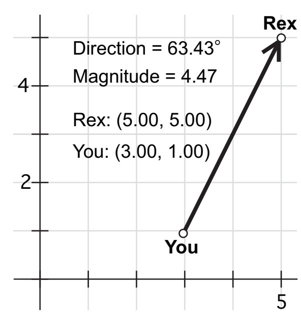
This activity is particularly effective with a student operating the computer and taking direction from you and from other students.

1. Open **Introduction to Vectors Present.gsp.** Describe the situation: Rex's leash is tied to a tree at the origin of an *xy*-coordinate system. Rex is pulling the leash tight as he excitedly waits for you to take him on a walk.

Q2 Get students to answer each of the questions on this page. Remind them periodically of the equivalence of the two different ways they are using to describe the same object: by components or by magnitude and direction.

Now it's time to untie the leash from the tree and take Rex for a walk.

2. On page 2 you've untied Rex from the tree, and he's pulling in a certain direction, no matter where you try to drag your end of the leash.



Q3 What are the components of vector **a**? How can you determine the components without moving point *You* to the origin?

Q4 Drag vector **a** around the screen and ask whether the vector changes as the coordinates of its head and tail change. Get students to explain why, no matter where you drag it, vector **a** is always the same vector. Encourage them to use both methods of describing the vector in their arguments.

Q5 On page 3, where will Rex be when you're at (80, 80)? How can you figure this out? Let the discussion develop until students find and understand a strategy.

Q6 On page 4 the objective is the same, but the available information is different. Encourage students to describe their strategy to each other.

Q7 What if your leash is twice as long, and Rex is still pulling in the same direction? Now where will Rex be when you're at (80, 80)? Answer for page 3 and page 4.

Vector Addition and Subtraction

You may have done an earlier activity in which you learned about vectors as you walked your dog Rex. In this activity you and Rex will go on some more walks together. Along the way you'll explore how to add and subtract vectors.

SKETCH AND INVESTIGATE

1. Open **Vector Add Subtract.gsp.**

Page 1 has two movable vectors, **a** and **b**. You and Rex begin your walk at home (the origin), heading first along a path the direction and magnitude of vector **a**, and then along another path the direction and magnitude of vector **b**.

Magnitude is another word for the *size* or *length* of a vector.

Q1 Drag the vectors to represent your walk. Find the coordinates of the ending point of your walk. Explain how you got your answer.

Q2 Some days Rex prefers a different route: **b**, then **a**. Where do you and Rex end up after taking this route?

Your walks in Q1 and Q2 could have taken more direct paths. Instead of following vectors **a** and **b**, you could have gone directly to the final destination along a single vector, **c**.

Q3 Choose **Vector** from the Custom Tools menu. This tool draws a vector with an arrow at its head. Use it to draw the vector **c** that would take you directly from home to your final destination. What are the coordinates of the head and tail of **c**?

Q4 On page 2, the coordinates of only one point are given. Assuming that you start at home, where do you end up? How did you get your answer?

2. Use the **Vector** tool to draw the vector representing the direct path from home to your final destination.

As before, you won't need to measure any new coordinates.

Q5 On page 3 are two identical pairs of vectors. Build one route to your destination with one set (**a**, then **b**), and then the other route with the other set (**b**, then **a**). For both routes, at what coordinates do you end up? Do the two routes take you to different final destinations? Is one route longer than the other?

3. Use the **Vector** tool to draw the direct path from home to your final destination.

Q6 All along, you've been combining vectors by putting the tail of the second vector on the head of the first. Would you get the correct destination if you put the head of the second vector on the head of the first? Why or why not?

VECTOR ADDITION

What you've just been doing is adding vectors! In step 2 you found a single vector, **c**, that got you to the same place as walking along **a**, then **b**. So we can say that $\mathbf{c} = \mathbf{a} + \mathbf{b}$.

The coordinate system made it easy to find the coordinates of **c**'s head. But if there is no coordinate grid, can you still draw $\mathbf{a} + \mathbf{b}$?

4. Go to page 4. What do you think $\mathbf{a} + \mathbf{b}$ will look like? Make a conjecture, and then press the button *Add a + b* to see.
5. What will $\mathbf{b} + \mathbf{a}$ look like? Make a conjecture, and then press *Add b + a* to see.

Q7 On your paper, describe vector addition in a way that someone familiar with the basics of vectors—but not yet vector addition—would understand. Make sure to use the words *head* and *tail.*

Q8 Addition of real numbers is *commutative* because $x + y = y + x$. (For example, $3 + 5 = 5 + 3$. Both operations give the answer 8.) Is vector addition commutative? In other words, does $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$? Justify your answer with a drawing.

Q9 The *zero vector,* **0**, is a vector with a magnitude of zero (basically, just a point). What must be true of two vectors whose sum is **0**?

VECTOR SUBTRACTION

Consider subtraction of real numbers. Recall that by definition, $8 - 5 = 8 + (-5) = 3$. *Subtracting a number is the same as adding its opposite.* The same is true with vectors: *Subtracting a vector is the same as adding its opposite.*

6. On page 5, experiment just as you did on the previous page.

Q10 On your paper, describe vector subtraction in a way that someone familiar with the basics of vectors—but not yet vector subtraction—would understand.

Q11 Is vector subtraction commutative? In other words, does $\mathbf{a} - \mathbf{b} = \mathbf{b} - \mathbf{a}$? Defend your answer with a drawing.

EXPLORE MORE

Q12 One way of describing a vector is to use its components: the coordinates of its head if its tail is at the origin. Assume that $\mathbf{a} = (a_1, a_2)$ and $\mathbf{b} = (b_1, b_2)$. If $\mathbf{a} + \mathbf{b} = \mathbf{c}$, what are the components of **c**? If $\mathbf{a} - \mathbf{b} = \mathbf{d}$, what are the components of **d**?

Objective: Students learn to add and subtract vectors graphically and algebraically, and investigate whether these two operations are commutative.

Student Audience: Algebra 1/Algebra 2

Prerequisites: Students should be familiar with the concept of vectors, and should be comfortable describing them using components or using magnitude and direction.

Sketchpad Level: Easy. Students work with a pre-made sketch.

Activity Time: 20–30 minutes. It may be possible to do the activity Introduction to Vectors: Walking Rex and this activity in one class period.

Setting: Paired/Individual Activity (use **Vector Add Subtract.gsp**) or Whole-Class Presentation (use **Vector Add Subtract Present.gsp**)

Related Activity: Introduction to Vectors: Walking Rex

SKETCH AND INVESTIGATE

Q1 The coordinates of the ending point are (7, 9). To find this, put **a**'s tail at the origin and **b**'s tail at **a**'s head. The coordinates of point *B*, **b**'s head, are the answer.

Q2 The answer is still (7, 9).

Q3 The head is at (7, 9) and the tail is at (0, 0).

Q4 You end up at (21, 10). To get there, put **b**'s tail at the origin and **a**'s tail at **b**'s head so that point *A*, whose coordinates are given, is the destination point.

Q5 Both routes take you to (19, 9). Put **a**'s tail at the origin this time and **b**'s tail at **a**'s head so that point *B*, whose coordinates are given, is the destination point. For the other route, put **b**'s tail at the origin and **a**'s tail at **b**'s head. The coordinates for both destinations are the same, so they take you and Rex to the same place. The length of each route is the sum of its component vectors. Since the two routes have the same components, they also have the same length.

Q6 You get an incorrect destination when you line up the head of one vector with the head of another because this violates the meaning of a vector. Vectors have a definite direction. If you line up their heads, you would be going against the direction of the second vector, which is sort of like going the wrong way down a one-way street.

VECTOR ADDITION

Q7 Line up the tail of the second vector with the head of the first vector. The sum is the vector from the first vector's tail to the second vector's head.

Q8 Yes, vector addition is commutative. The drawing on the first page of the activity demonstrates this: Whether you add the second vector to the first or the first to the second, you end up in the same place.

Q9 Two vectors whose sum is zero have the same magnitude but face in opposite directions.

VECTOR SUBTRACTION

Q10 Vector subtraction is just like vector addition except that what is added is the *opposite* of the second vector (which we'll call the *opposite vector* here). Thus, line up the head of the first vector with the tail of the opposite vector. The difference between the two vectors is represented by the vector connecting the tail of the first vector to the head of the second vector's opposite. You are adding the opposite of the second vector to the first vector.

Q11 No, vector subtraction isn't commutative. In fact, the difference of two vectors is the opposite of the difference taken in the reverse order.

EXPLORE MORE

Q12 The components of vector **c** are $(a_1 + b_1, a_2 + b_2)$. The components of vector **d** are $(a_1 - b_1, a_2 - b_2)$.

Substitute the name of one of your students.

In this activity a student, Yolanda, and her dog, Rex, will take some walks together, and in the process explore how to add and subtract vectors.

VECTOR ADDITION

This activity is most effective if a student operates the computer and takes direction from you and from other students.

1. Open **Vector Add Subtract Present.gsp.** Page 1 has two movable vectors, **a** and **b**. Yolanda and Rex begin their walk at home (the origin), heading first along a path the direction and magnitude of vector **a**, and then along another path the direction and magnitude of vector **b**.

Q1 Drag the vectors so they show this walk. Find the coordinates of the ending point of the walk. Have students explain how they got their answers.

Q2 Some days Rex prefers a different route: **b**, then **a**. Rearrange the vectors to represent this route. Where do they end up?

Q3 On page 2 there are two vectors, but only one shows its coordinates. If Yolanda and Rex start at home, where do they end up? How did you have to arrange the vectors?

Choose the **Vector** custom tool by pressing and holding the **Custom** tools icon, and selecting **Vector** from the menu that appears. Use it by clicking first the tail, then the head of the desired vector.

2. Use the **Vector** custom tool to draw the vector representing the direct path from Yolanda's home to the final destination.

Q4 On page 3 are two identical pairs of vectors. Build one route to the destination with one set (**a**, then **b**), and then the other route with the other set (**b**, then **a**). Where does each route end up? Is one route longer than the other?

3. Use the **Vector** tool to draw the direct path from Yolanda's home to her destination.

The new vector you just constructed is the sum of the other two vectors.

Q5 Addition of real numbers is *commutative* because $x + y = y + x$. (For example, $3 + 5 = 5 + 3 = 8$.) Is vector addition commutative? Does $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$? Justify your answer based on the vector operations you've done.

VECTOR SUBTRACTION

Recall that subtracting a number is the same as adding its opposite. The same is true with vectors: *Subtracting a vector is the same as adding its opposite.*

Q6 How can you make the opposite of a vector? (Keep the same magnitude, but reverse the direction.)

4. On page 4, experiment with vector subtraction. Use the button to find the opposite of **b**. Then subtract the vectors by adding **a** and the opposite of **b**.

Q7 Is vector subtraction commutative? Use the sketch to investigate.

Solving Systems Using Matrices

One of the most common applications of matrices is to solve a system of linear equations. For a solution to be possible, the number of equations must be greater than or equal to the number of unknown values. In this activity you'll solve systems of two equations with two unknowns.

INTERSECTION OF LINES

1. Open **Matrix Solution.gsp**. The sketch contains two lines in the coordinate plane, along with their equations in standard form ($ax + by + c = 0$). You can control the equations and the lines by editing the parameters a_1, b_1, c_1, a_2, b_2, and c_2.

You have seen problems like this before. This is a system of two equations with two unknowns, x and y. The coordinates of the intersection point are the solution. This time you will use matrices to solve the system. First you have to express the system of equations as a matrix equation:

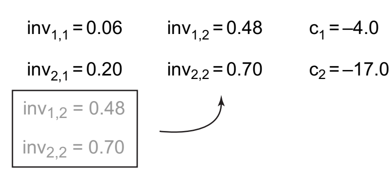
$$\begin{matrix} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{matrix} \Rightarrow \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

Q1 In this equation there are only two unknowns. What are they?

The next step is to make a formula by solving for the unknowns:

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \Rightarrow \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}^{-1} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

2. This formula requires an inverse matrix. Arrange parameters a_1, b_1, a_2, and b_2 into a matrix as shown here. Press and hold the **Custom** tools icon, and choose **2D Inverse.** Click the four matrix elements, working down the left column first, and then down the right column: a_1, a_2, b_1, b_2.



3. The result of using the tool is the inverse matrix. It appears as a single column. Drag the two bottom elements up and to the right to form the second column. Arrange parameters c_1 and c_2 as a column vector on the right side of the inverse vector.

4. Press and hold the **Custom** tool button, and choose **2D Matrix * Vector.** Select the elements of the matrix formula. Work down the left matrix column first, then down the right column, and finally down the column vector. The output is the solution. The results of this calculation will be labeled *c[1]′* and *c[2]′*. Use the **Text** tool to change the labels to x and y.

5. Select the coordinates in order. Choose **Graph | Plot As (x, y).**

Q2 Does the plotted point fall on the intersection of the lines?

Q3 By changing the equation parameters, find the solution to this system:

$$3x + 10y = -8$$
$$-4x + y = -18$$

Q4 Edit the parameters again to form the system shown below. The system has no solution. How can you tell that by examining the matrix equation?

$$-12x + 9y = -4$$
$$4x - 3y = -17$$

FITTING A LINE

6. On page 2 there are two points and their coordinates.

As you know, there is exactly one line through both of these points. Your objective is to find the slope-intercept equation of that line. Both points satisfy the equation, so that gives you this system:

$$mx_P + b = y_P$$
$$mx_Q + b = y_Q$$

See those two 1's at the edge of the screen? You will need them when you form your matrix.

Q5 This time you know *x* and *y* in both equations. The unknown values are *m* and *b*. Rewrite this system of equations as a matrix equation. Then rewrite that as a matrix formula for the unknowns.

7. Repeat the steps from the earlier section to find *m* and *b*.

8. Choose **Graph | Plot New Function.** For the function definition, enter $mx + b$.

Q6 If the two points have the same *x*-coordinate, it is still possible to draw the line between them, but it is not possible to write the equation in slope-intercept form. Drag one of the points so that it is directly above the other. What happens to the function plot? Explain why the matrix solution fails.

EXPLORE MORE

Page 3 is identical to page 2. Use a similar procedure to fit the two points with the graph of an equation in this form:

$$a\cos x + b\sin x = y$$

Objective: Students model a system of equations as a single matrix equation. They then use the inverse matrix to find the solution.

Student Audience: Algebra 2/Precalculus

Prerequisites: Students should have a basic understanding of matrix algebra. They will not have to calculate an inverse matrix themselves, but they will have to use one.

Sketchpad Level: Intermediate. Students will do most of the work with the aid of custom tools.

Activity Time: 20–30 minutes

Setting: Paired/Individual Activity (use **Matrix Solution.gsp**) or Whole-Class Presentation (use **Matrix Solution Present.gsp**)

Although students should learn how to compute the inverse of a matrix, that task would make this activity more tedious, and it would greatly increase the likelihood of a blunder. For those reasons the partially completed document has custom tools for computing the matrix and for multiplying a matrix by a vector.

INTERSECTION OF LINES

Q1 The unknowns are x and y.

Q2 The plotted point falls on the intersection.

Q3 The solution to the given system is $x = 4$, $y = -2$.

Q4 This system is represented by the following matrix equation:

$$\begin{bmatrix} -12 & 9 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ -17 \end{bmatrix}$$

The matrix is singular. Students may see that the top row is a multiple of the bottom, or they may see that the determinant is zero. This means that it has no inverse, and the system has no solution. Students may also observe that the lines are parallel.

FITTING A LINE

Q5 Be sure to check for understanding at this point. There may be a tendency for students to try to force the matrix equation into the same form that was used in the previous section, but this situation is very different.

$$\begin{matrix} mx_P + b = y_P \\ mx_Q + b = y_Q \end{matrix} \Rightarrow \begin{bmatrix} x_P & 1 \\ x_Q & 1 \end{bmatrix}\begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} y_P \\ y_Q \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_P & 1 \\ x_Q & 1 \end{bmatrix}^{-1}\begin{bmatrix} y_P \\ y_Q \end{bmatrix} = \begin{bmatrix} m \\ b \end{bmatrix}$$

Q6 This is the same problem that occurred in Q3. The matrix is singular because the top and bottom rows are identical. Therefore, it has no inverse, and the system has no solution. In this case no solution means only that there is no slope-intercept equation. The line may be possible, but its equation would have to be in a different form.

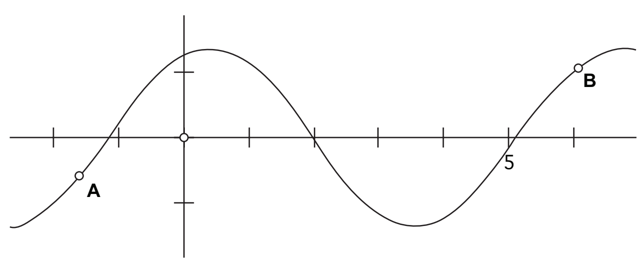
EXPLORE MORE

The trick to this extension is to mentally separate the known values and the unknown. Since the x- and y-coordinates of two points are known, you can also regard the sine and cosine parts of the equation as known values. Use the Sketchpad Calculator to compute $\cos x_P$, $\sin x_P$, $\cos x_Q$, and $\sin x_Q$. Then arrange those calculations into a matrix as shown:

$$\begin{matrix} a\cos x_P + b\sin x_P = y_P \\ a\cos x_Q + b\sin x_Q = y_Q \end{matrix} \Rightarrow \begin{bmatrix} \cos x_P & \sin x_P \\ \cos x_Q & \sin x_Q \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} y_P \\ y_Q \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos x_P & \sin x_P \\ \cos x_Q & \sin x_Q \end{bmatrix}^{-1}\begin{bmatrix} y_P \\ y_Q \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

After solving for a and b, define and plot the function $f(x) = a\cos x + b\sin x$. The result creates a much more interesting graph than the linear function although the procedure is nearly the same.



In this presentation you'll show students how to solve a system of equations by forming a matrix of coefficients and using the inverse matrix to compute the solution.

The presentation will be most effective if you have a student operate the computer.

1. Open **Matrix Solution Present.gsp.** The six red and blue parameters control the two linear equations above them. They also control the lines themselves. Change some of the parameters to show the relationship.

Q1 The solution to this system of linear equations corresponds to the intersection of the lines. How can you represent this system as a single matrix equation? After suitable input from students, press *Matrix* to show the answer.

Q2 The variables x and y are the only unknowns here. How can you rewrite the equation to solve for x and y? Press the *Inverse* button to show the formula.

At this time you may wish to have the class derive the inverse of the matrix. If so, simplify the task by making all the parameters integers.

2. Press *Compute* to show the inverse matrix, and press *Solution* to show the plotted point.
3. Before moving to the next part, press *Reset* and briefly go through the steps again.
4. Go to the Fitting a Line page. This time there are two points and no line.
5. Press *Coordinates*. Drag the points around and explain that the objective is to find the slope-intercept equation of the line joining these two points.

Q3 What two equations can you write using the given information? Tell students to take their time on this part. They may try to force the equations into the same form as in the previous section. Get them to use the slope-intercept form. After a suitable period of discussion, press *Show Equations*.

6. Continue through the buttons in order, showing the matrix, its inverse, and finally the solution. Urge students to keep in mind which values they are looking for. Rather than x and y, they are now looking for m and b.

Q4 After showing the solution, drag one of the points so that it is directly above the other. Why does the solution line disappear? Students may know that the slope-intercept equation cannot represent a vertical line, but how can they tell from the matrix? Use the Calculator to show that the determinant is zero.

Q5 Go back to the Intersection of Lines page. Adjust the parameters to form the equations below. Why is there no solution? Again, show that the determinant is zero.

$$-12x + 9y = -4$$

$$4x - 3y = -17$$