Project Ideas

You can use the projects described below to extend your understanding of algebra. These projects are suitable for in-class presentations, for research papers, or for personal exploration.

FUNCTIONS

1. In the activity Function Composition with Dynagraphs, you learned how to use dynagraphs to model composite functions. How would this work with Cartesian graphs? To find out, open the sketch **Cartesian Composition.gsp** and follow the directions.

2. In Function Composition with Dynagraphs (specifically, Q2 and Q5), you saw that for some functions f and g, $f(g(x))$ always equals $g(f(x))$, while for some it doesn't. Do some research and find out what functions f and g are called if $f(g(x))$ does equal $g(f(x))$ for all values of x. What do these pairs of functions look like when modeled using Dynagraphs? What do they look like plotted in the Cartesian plane?

3. One of the best-known iteration rules in mathematics produces the famous Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, . . . Each term in the sequence (after the second) is the sum of the two preceding terms. This sequence shows up in many different areas of mathematics and the natural world.

   You can look at this sequence in terms of coordinates, taking each adjacent pair of terms as the coordinates of a point: (1, 1), (1, 2), (2, 3), (3, 5), (5, 8), . . . Can you adjust your iteration rule from Functions Again and Again to produce these points? What shape does this sequence of points yield on the graph? How does this shape vary when you move the starting point? Can you find other ways of representing the Fibonacci sequence in Sketchpad?

4. Open **Supplemental Sketches | Arithmetic Triangle.gsp.** The arithmetic triangle, also known as Pascal's triangle, is usually associated with the coefficients of binomial expansion; but over the centuries, many other mathematical patterns have been discovered in the figure. In this sketch the triangle connects number theory with fractal geometry. Each cell is colored red or blue depending on whether its value is divisible by the parameter *Divisor*. Change the *Divisor* and try to predict and explain the patterns that appear.

   You can derive one row of numbers from the row above it. Using remainder theory you can also find the remainders in one row knowing only the remainders in the row above. Hide the values and show the remainders. This simplifies the analysis by limiting the size of the numbers.

SYSTEMS

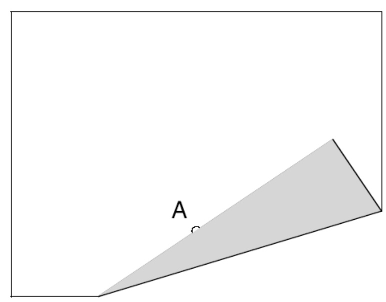
5. In the activity Solving Systems of Equations you used two linear equations to represent the price structure of two moving companies. You can also use a similar pair of linear equations to represent two runners starting at different positions and running at different speeds. Write the equations to represent Yvonne (who starts at the 2 mile mark of the race course and runs at 10 miles per hour) and Tonya (who starts at the 1 mile mark and runs at 12 miles per hour). Graph your equations and use the graph to determine when Tonya will catch Yvonne.

6. In a famous race from long ago, a Tortoise was racing Achilles in the Greek philosopher Zeno's famous paradox. Look up "Zeno's Paradox" in the library or on the web and present a report on your findings. Create a sketch using Movement buttons to model the race described in the paradox.

QUADRATIC FUNCTIONS

7. Given any two points, it's possible to construct the unique line containing them. In fact, Sketchpad's Line tool does just that. How many points are required to define a unique parabola? If you have that number of points, can you arrange them in such a way that they don't define any parabola? Explore these questions. As an extra challenge, create a custom tool called Parabola that constructs a parabola when you click on the proper number of arbitrary points.

8. Make a parabolic envelope as follows: Mark a point *A* about 1 inch from the bottom of a blank piece of paper. As shown at right, fold the paper so that a point on the bottom edge lands directly onto point *A*. Make a sharp crease to keep a record of this fold, then unfold it. Fold the paper along a new crease so that a different point on the bottom edge lands on point *A*. Repeat the process many times. Eventually, the creases will outline a parabola. Can you explain why this process works? Can you model this paper-folding technique in Sketchpad? See the activity The Folded Rectangle Construction from the Key Curriculum Press book *Exploring Conic Sections with The Geometer's Sketchpad* for more information.



9. Open **Supplemental Sketches | Conic Sections.gsp.** The sketch shows a cone being cut by a plane, and allows you to control the position and angle of the plane and to view the cone from various angles. Experiment with the controls to cut the cone in various ways and to view the shapes generated. Under what circumstances is the shape a parabola? How must you adjust the plane to

generate an ellipse, a circle, or a hyperbola? How can you make the ellipse long and narrow? How can you produce two crossed lines? How can you produce a single line?

ALGEBRAIC TRANSFORMATIONS

10. Sketchpad uses four built-in geometric transformations: translation, rotation, dilation, and reflection. This chapter has activities for all of these except dilation. Instead of activities for dilation, it has activities for stretching and shrinking. What is the relationship between the geometric transformation of dilation, and the algebraic transformations of stretching and shrinking? Create a sketch (containing both kinds of transformations) to show the how these geometric and algebraic transformations are similar and how they are different.

11. This chapter has an activity on rotating coordinates, but no activity on rotating function plots. Plot an interesting function and construct a point on the function plot. Then rotate the point using the methods in the activity Rotating Coordinates, and find the locus of the rotated point as the original point moves along the function plot. Investigate the algebraic characteristics of the rotated function plot. What do you observe? What conclusions can you draw for rotations by 90°, 180°, and 270°?

12. In the activity Rotating Coordinates, the *polar grid* and *polar coordinates* were introduced. Explore graphing in the polar grid as follows: In a new sketch, choose **Graph | Grid Form | Polar Grid.** Choose **Graph | Plot New Function,** enter 3, and click OK. The resulting graph is $f(\theta) = 3$. Why does it look the way it does? Try plotting $f(\theta) = \theta$ or other functions of θ.

13. Create another polar grid in a new sketch. Choose **Graph | Plot New Function,** and in the New Function dialog box choose **θ = f (r)** from the Equation pop-up menu. Then type 45 to plot the function $f(r) = 45$. Explore the plots of other functions of r. In all cases, try to explain why the graph makes sense given what you know about polar coordinates.

OTHER FUNCTIONS

14. In this chapter are activities related to inverses of exponential functions (logarithmic functions) and activities related to inverses of quadratic functions (square root functions). What do the inverses of some of the other functions in this chapter look like? Investigate inverses of square root functions or inverses of rational functions. Use Sketchpad to produce graphs of these inverses, and present your conclusions to your teacher or to your class.

15. Graph and compare the power function $f(x) = x^2$ and the exponential function $g(x) = 2^x$. When $x = 0$, $g(x)$ is greater than $f(x)$. As x increases, $f(x)$ becomes bigger; then $g(x)$ takes the lead again and never looks back.

    Can a power function win this race with a handicap? Let $f(x) = x^3$ and $g(x) = 1.5^x$. Try larger exponents for the power function and smaller bases (but greater than one) for the exponential function.

TRIGONOMETRIC FUNCTIONS

16. In this chapter you explore the sine, cosine, and tangent functions. How do the inverses of these functions behave? Create a sketch showing these inverse functions, and explore how the inverse of each of these functions is similar to the original function and how it is different. Present your sketch and your conclusions.
17. Plot several trigonometric functions using polar coordinates. Interesting ones to try include $f(\theta) = \sin(k\theta)$ for various integer values of k. You might also try $f(\theta) = 2 / \sin(\theta)$. Experiment with some others of your own.
18. Resection is a surveying and mapping procedure for finding your own position without measuring any distances. Suppose that you can see certain landmarks having known coordinates. You cannot record bearings — a compass is not precise enough — but you can measure angles between the landmarks. How many landmarks do you need? Derive the formulas. Create a simulation in Sketchpad.

PROBABILITY AND DATA

19. Collect some interesting data from your classmates, from your school, or from the web. Plot the data in Sketchpad, and then use the tools from Fitting Functions to Data to try fitting various functions to the data. Prepare a report of your findings, and try to explain why the particular function you chose is a reasonable choice for the data you collected.
20. Open **Supplemental Tools | Function Fitters.gsp** and experiment with each of the tools provided there. Find images from the web that contain shapes that you can fit with polynomials, with exponential functions, or with rational functions. Paste the images into Sketchpad, and use the function fitter tools to fit a function to the shape in the image and find an equation for the shape.