Point Field

Here is the standard form for the equation of a line:

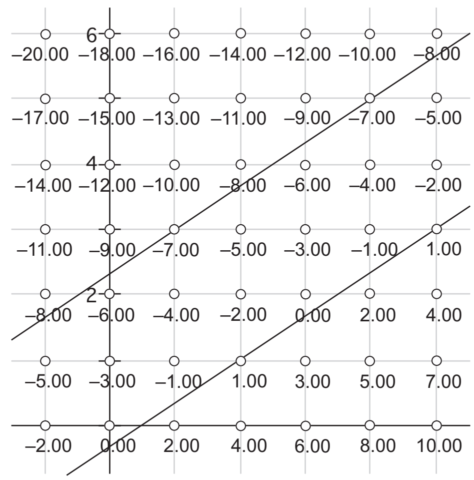
$$ax + by = c$$

A good way to understand the equation is to first consider only the left side, $ax + by$. This expression is called a *linear combination* of x and y.

Suppose that you are given certain values for the coefficients a and b. Any point in the coordinate plane has exactly one pair of coordinates (x, y). It follows that the linear combination $ax + by$ has exactly one value at any point.

GETTING STARTED

1. Open **Point Field.gsp.**



On the left side are two parameters, a and b. They control the coefficients of the expression $ax + by$. The points in the field have integer coordinates. Below each point is a value representing $ax + by$ evaluated at that point. The green point is a free point. You can drag it anywhere on the screen and use it as a probe.

You can also change a parameter by selecting it and pressing the + or − key.

2. Experiment with the parameters. Change a parameter by double-clicking it and entering a different value. Observe the effect on the point field.

3. Set the parameters to $a = 2$ and $b = -3$. Find two points that have the value 1.00 displayed beneath them. Use the **Line** tool to construct a line between these two points.

Q1 Can you find any other points that fall on this same line? What are the values of x and y associated with those points?

4. Find two points that have the value -7.00 beneath them. Construct a line between them.

Q2 What values of x and y are associated with the other points falling on this line?

Q3 What are the equations of these two lines? Your answers to Q1 and Q2 can help you with this one.

Q4 The lines cut the point field into three regions. Inspect the points that fall above both lines, below both lines, and between the lines. What pattern do you observe in the values associated with each set of points?

To measure the slopes, select both lines and choose **Measure | Slope.**

5. Measure the slopes of the two lines.

Q5 From the slope measurements and other observations, what can you say about the relationship between these lines? How are their slopes related to parameters *a* and *b*?

6. Find several other pairs of points with matching values and construct lines between them.

Q6 What general statement can you make about the family of lines in the form $2x - 3y = c$?

INVESTIGATE ANOTHER FIELD

To hide the lines and measurements, select them and choose **Display | Hide.**

7. Hide all of the lines and measurements that you constructed in the previous section.

8. Change the parameters to $a = 12$ and $b = 20$. Use the point field to construct lines $12x + 20y = -36$ and $12x + 20y = 56$.

Q7 Inspect the values on the points that fall above both lines, below both lines, and between the lines. What pattern do you observe in the values associated with each set of points? Compare this with your answer to Q4.

EXPLORE MORE

Experiment with changing the parameters, and answer the questions below.

Q8 How is it possible to make points of equal value fall on a horizontal line? What about a vertical line?

Q9 Is it possible to set the parameters so that there are three or more points that have the same value but do not lie in a line? Explain why.

Q10 Can you set the parameters so that there are no two points in the field with the same value? Explain how to do it, or why you cannot do it.

Q11 Construct a line through two of the field points that do not have the same value. Examine the values of all of the points that fall on this line. What pattern do you see?

Objective: Students use a prepared point field to investigate linear combinations in the form $ax + by$.

Student Audience: Algebra 1/Algebra 2

Prerequisites: Students should be familiar with the concept of a line representing the set of points whose coordinates satisfy a given equation. Ideally, use this activity shortly after they have learned the standard form.

Sketchpad Level: Easy

Activity Time: 30–40 minutes

Setting: Paired/Individual Activity (use **Point Field.gsp**) or Whole-Class Presentation (use **Point Field Present.gsp**)

GETTING STARTED

Q1 Several points fall on this line. The points all have a value of 1.00.

Q2 The points all have a value of -7.00.

Q3 The equations are $2x - 3y = 1$ and $2x - 3y = -7$.

Q4 Above both lines, $2x - 3y < -7$. Below both lines, $1 < 2x - 3y$. Between the lines, $-7 < 2x - 3y < 1$.

Q5 The slope measurements will both be 0.67, which suggests that the lines are parallel. In fact, they are, and the slope is equal to $-a/b$.

Q6 All lines in this family are parallel.

INVESTIGATE ANOTHER FIELD

Q7 At each of the points above both lines, $56 < 12x + 20y$. Below both lines, $12x + 20y < -36$. Between the two lines $-36 < 12x + 20y < 56$. Students may notice that in Q4, the larger numbers fell below the lines, but in this case, they are above. This difference is caused by the change in the sign of parameter b.

EXPLORE MORE

Q8 To make a horizontal line, set $a = 0$. To make a vertical line, set $b = 0$.

Q9 If both parameters are set to zero, then the linear combination will be zero for every point in the plane.

Q10 The field has a limited range and it has only points with integer coordinates, so this is possible. One good strategy is to pick rather large, relatively prime numbers for the coefficients. For instance, setting $a = 19$ and $b = 20$ works.

A more interesting strategy would be to use one rational coefficient and one irrational, e.g., $a = 1$, $b = \pi$. In that case, disregarding the rounding limitations, no two points with integer coordinates would have the same value, even if the field could be extended to cover the entire plane.

Q11 The values for consecutive points on the line are in arithmetic progression, and the points themselves are uniformly spaced.

The document **Point Field.gsp** has a few features that were not used in the activity. Press the *Show Controls* button to see them. The density of the field can be changed, and the field limits can be moved.

The second page of the document has both the point field and a linear inequality graph. This may be useful for showing that an inequality region is filled with points that satisfy it.

Begin by taking a few minutes to explain the concept of a linear combination in the form $ax + by$. Given parameters a and b, every point in the xy plane has one value defined by this linear combination. Stress the fact that a linear combination is not an equation.

1. Open **Point Field Present.gsp.** Parameters a and b define the linear combination. A field of points covers part of the coordinate plane. The points have integer coordinates, and the linear combination is evaluated at each point.
2. Change the parameters to show how the point values respond.
3. Set the parameters to $a = 2$ and $b = -3$.
4. Find two points with the value 1.00 and construct a line between them.

Q1 Are there any other points falling on this line? What are their values? (Any other point falling on this line will have a value of 1.00.)

5. Find two points with the value -7.00 and construct a line between them.

Q2 What are the values of other points falling on this line? (-7.00)

Q3 The lines cut the point field into three regions. What pattern do you observe in the values associated with each set of points? (For points above both lines, $ax + by < -7$. Below both lines, $ax + by > 1$. Between the lines, $-7 < ax + by < 1$.)

Q4 What are the equations of these two lines?

(The equations are $ax + by = 1$ and $ax + by = -7$. Be sure to bring out the connection between the observations and the equation. The first line, for example, includes all points for which the linear combination is equal to one, and only those points, hence the equation.)

Q5 The lines appear to be parallel. How can we confirm that?

(Both equations are in standard form, so the slope is $-a/b = 2/3$ for both equations. An alternate way to show this is to select both lines and choose **Measure | Slope.**)

6. Hide the lines, change the parameter definitions to some other arbitrary numbers, and repeat the investigation.

The second page has the same point field along with the graph of a linear inequality. The inequality uses the same linear combination as the point field. Use this page to show that an inequality region is filled with points that satisfy the inequality.